
To boldly go where no
man has gone before

The Scales of Complexity: What does the Devil's Advocate say?

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Celebrating Germinal Cocho's 80th Birthday

Who is the Devil's Advocate?

Promotor Fidei or Advocatus Diaboli

A popular title given to one of the most important officers of the Sacred Congregation of Rites, established in 1587, by Sixtus V, to deal juridically with processes of beatification and canonization. His official title is Promoter of the Faith *(Promotor Fidei)*. His duty requires him to prepare in writing all possible arguments, even at times seemingly slight, against the raising of any one to the honours of the altar. The interest and honour of the Church are concerned in preventing any one from receiving those honours whose death is not juridically proved to have been "precious in the sight of God". In counterposition to the Advocatus Dei or Promoter of the Cause.

Wiki

In common parlance, a **devil's advocate** is someone who, given a certain argument, takes a position they do not necessarily agree with (**or simply an alternative position from the accepted norm**), for the sake of debate or to explore the thought further. In taking this position, the individual taking on the devil's advocate role seeks to engage others in an argumentative discussion process. The purpose of such a process is typically to test the quality of the original argument and identify weaknesses in its structure, and to use such information to either improve or abandon the original, opposing position. It can also refer to someone who takes a stance that is seen as unpopular or unconventional, but is actually another way of arguing a much more conventional stance.

The candidates for canonization:

Building Blocks

The Proposed Miracles of Criticality - "The Edge of Chaos"

Species extinctions

Electromagnetism

Animal foraging

City populations

Income distribution

Fluids

Sandpiles

Word distributions

$$f(x) \sim x^{-\beta}$$

Earthquakes

Ferromagnets

Gravity

Metabolic rates

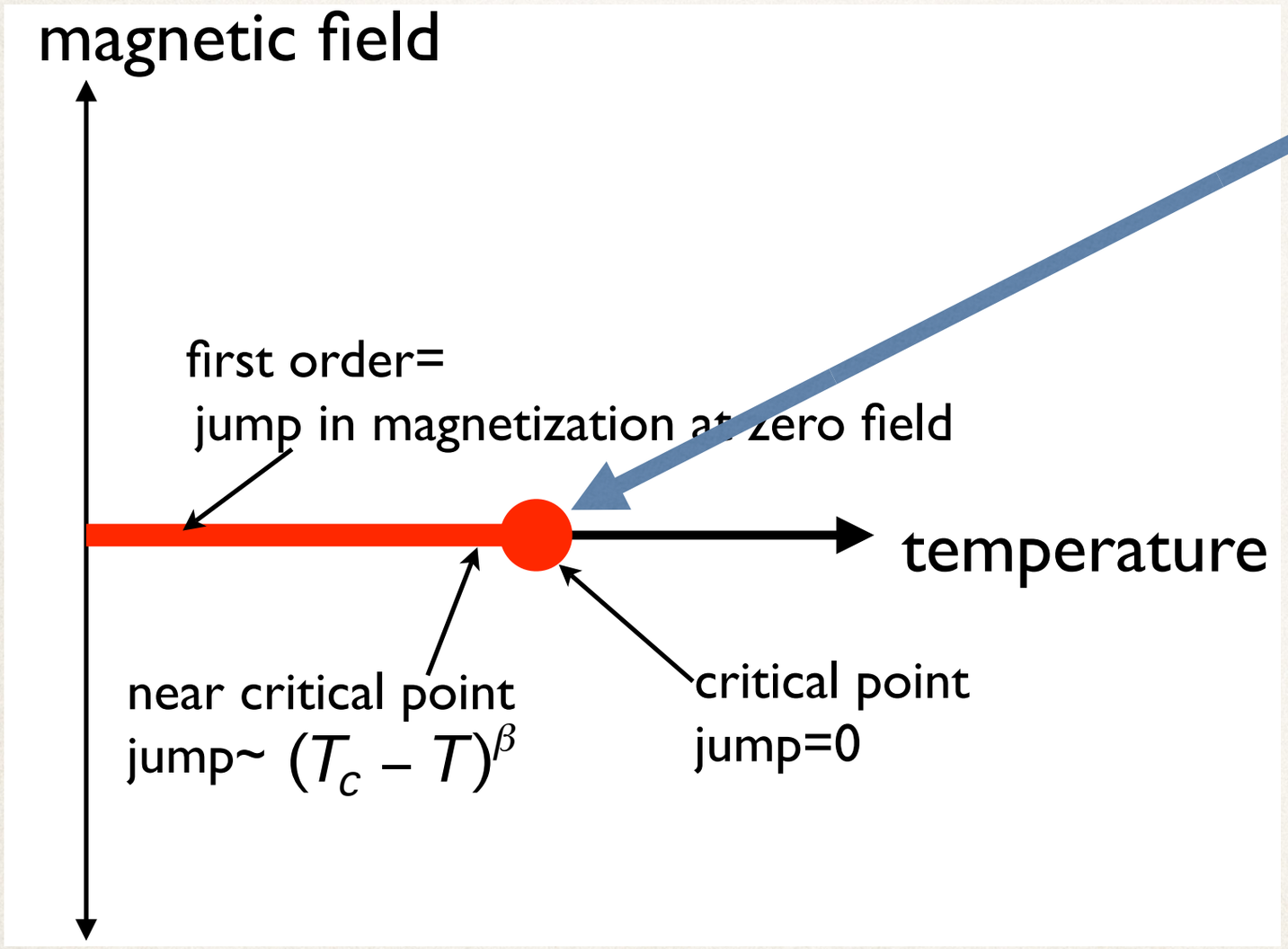
Prices

Impact of scientific articles

Where did it all begin?

Let's think of ferromagnets…

Phenomenology of the critical point



Saints van der Waals, Gibbs, Ehrenfest,…

The Miracles of the Critical point

- scaling
- scale invariance
- universality

$$\chi \sim t^{-\gamma} \qquad H = 0$$
$$\xi \sim t^{-\nu} \qquad H = 0$$
$$C \sim t^{-\alpha} \qquad H = 0$$
$$H \sim \varphi^{\delta} \qquad t = 0$$
$$\varphi \sim t^{\beta} \qquad H = 0$$
$$G(r) \sim r^{-(d-2-\eta)} \qquad t = 0,\ H = 0$$

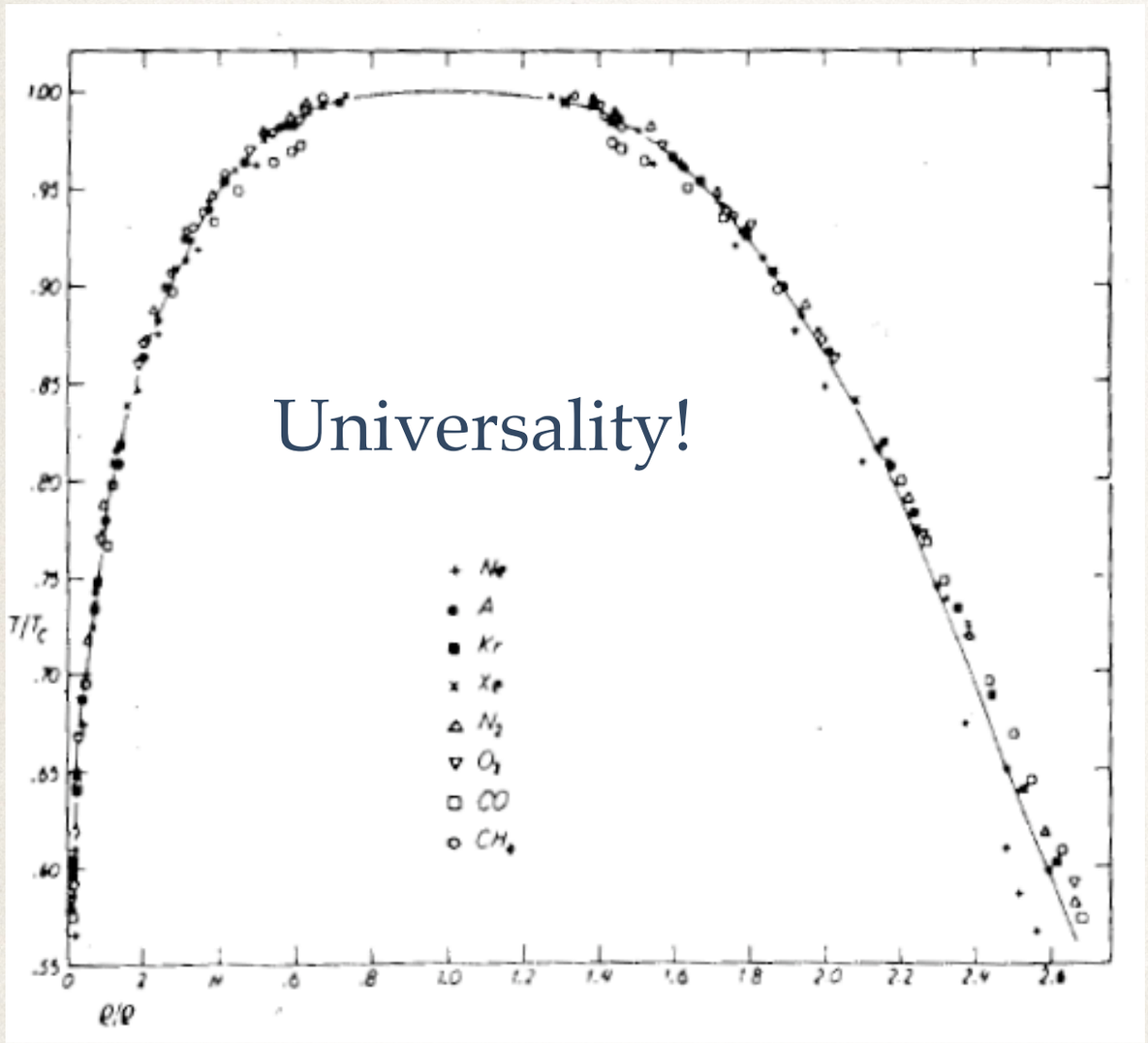
$$\alpha + 2\beta + \gamma = 2$$ Scaling laws

Scaling

$$H = f(t, \varphi) \quad \rightarrow \quad H = \varphi^{\delta} f(t/\varphi^{1/\beta})$$

Great theory shame about experiment!

Saint Landau's (mean field) theory $\gamma = 1, \quad \nu = 1/2, \quad \alpha = 0, \quad \delta = 3, \quad \beta = 1/2, \quad \eta = 0$



Saint Guggenheim, 1945

Experiment showed beta closer to 1/3 than 1/2 in 3D

Exact models and series expansions also showed that mean field theory was wrong. So what was wrong?

Fluctuations!

In the vicinity of the critical point many degrees of freedom are strongly coupled

Coarse graining

One of the most important miracles in all of science

- **Mapping systems with many degrees of freedom to one with fewer…**
  - Mechanics - e.g., rigid bodies; planetary motion,…
  - Statistical mechanics - e.g., kinetic theory, thermodynamics,…
  - Genetics - e.g., genotype-phenotype map; nucleotides to genes,…
  - Quantum field theory - e.g., renormalisation, bound states,…
  - Statistical mechanics/field theory - e.g., block spinning, majority rule,…

Single versus iterative coarse grainings…
maps from one system to "another"

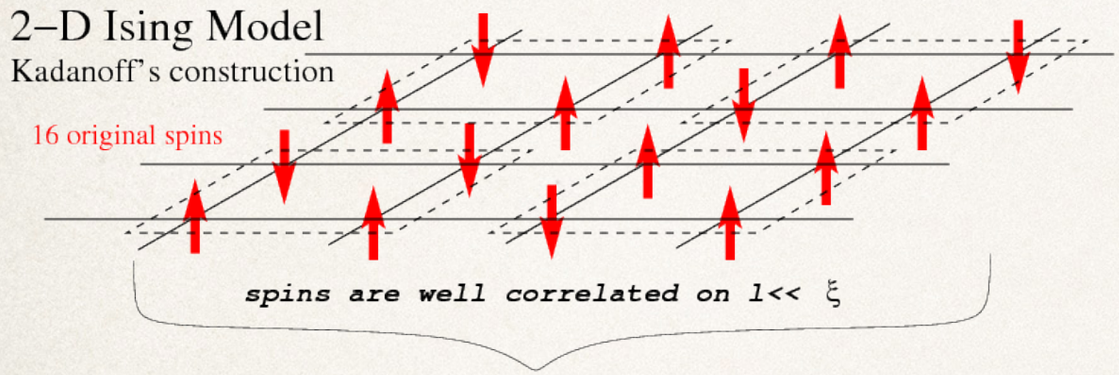
Saints Boltzmann, Migdal and Kadanoff, Mendel,…

Effective Theories

Saint Wilson's Renormalization Group:

Explains Scaling, Scale invariance and Universality!

And so did the Field theoretic RG - Saints Stuckelburg, Bogoliubov, Shirkov, Gell-Mann, Low,…



$$\mathcal{H} = -(J/2T)\sum_{i,j}\sigma_i\sigma_j - (H/T)\sum_i \sigma_i + E$$

Map from one lattice to another: $\mathcal{R}_a \quad \xi \to \xi/2$

$$\mathcal{H}(T,H,E) \to \mathcal{H}(T',H',E')$$

Map is on the space of theories

No guarantee the theory will "renormalize"

$$Z(T,H,E,a,N) = Z(T',H',E',a,N/2)$$
$$F(T,H,a,N) = F(T',H',a,N/2) + G(T,H)$$
$$\Gamma^{(N)}(T,H,a,N) = \Gamma^{(N)}(T',H',a,N/2)$$

for 1D

$$\mathcal{R}_a\mathcal{R}_b = \mathcal{R}_{ab}$$ Maps form a (semi)-group

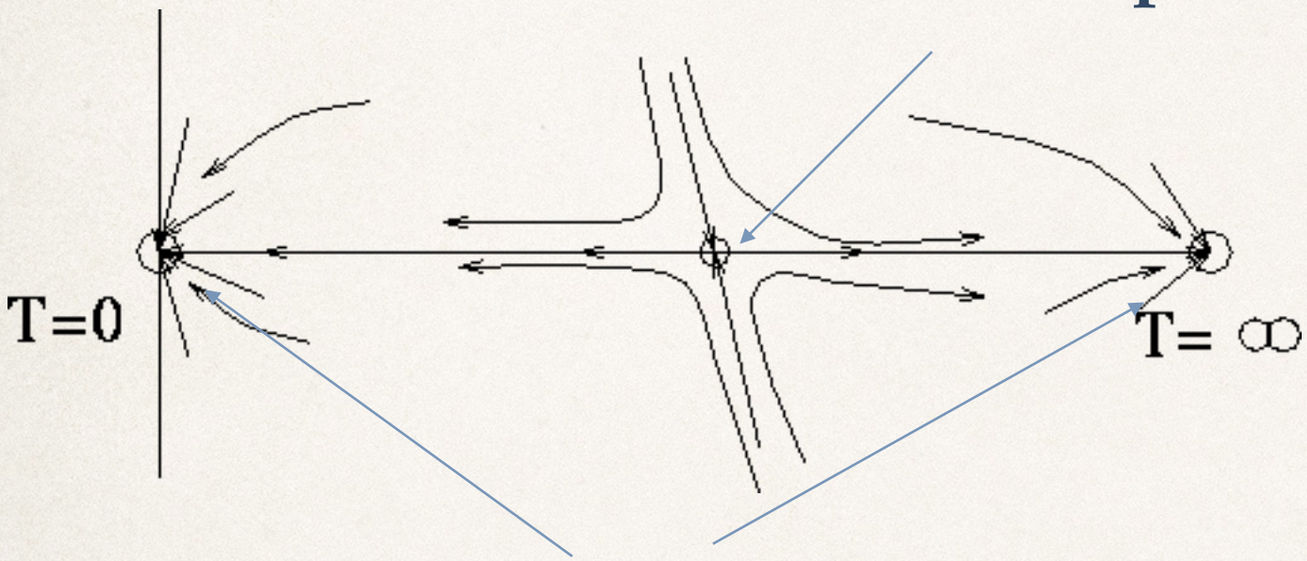
Iterate $H_n = \mathcal{R}_2^n H \quad T_n = \mathcal{R}_2^n T$ and look for fixed points: $T = T^*$ and $H = H^* \; \xi = \infty, \quad \xi = 0$

Linearize the transformation near the fixed point: $\mathcal{R}_b(T,H) \equiv (t',h') \sim (b^{y^t}t, b^{y^h}h)$

Critical exponents are related to the eigenvalues of the linearised RG transformation, e.g. $\nu = 1/y^t$.

Criticality is "special" and "boring"
(phenomenologically poor)

Critical point - unstable to relevant H and T parameters, sub-manifold in the space of parameters



i.e., Other fixed points of the RG

Crossover phenomena

In the vicinity of the critical point for a ferromagnet (and many others) there are **only two scales** - a microscopic lattice scale and the correlation length. When the correlation length is much bigger than the microscopic scale then we have **universality**. But, there are **always other scales…**

Crossovers

O'Connor, Denjoe, and C. R. Stephens.
"Renormalization group theory of crossovers."
Physics Reports 363.4 (2002): 425-545.

For a general scaling function $\mathcal{F}(x, y, z, ...)$ where $x = g^{\phi}/t$ or $x = g\xi$ where g is a generic anisotropy parameter, then:

In the isotropic limit: $x \to 0 \quad \mathcal{F} \to x^{a}\mathcal{A}(y, z, ...)$ ***The scaling function A contains a singularity***

In the anisotropic limit: $x \to x_s \quad \mathcal{F} \to (x - x_s)^{a_p - a}\mathcal{A}_p(y, z, ...)$

A richer phenomenology than standard criticality
Two different points of scale invariance
Two different fixed points of the RG!
A linearisation around one fixed point cannot access the other

Some examples of g: System size L, dipolar coupling, temperature in quantum ferromagnets, distance to surface for surface/bulk (wetting), spin anisotropy, kinematic heterogeneity, etc.

Critical phenomena in a "box"

Environmentally Friendly Renormalization

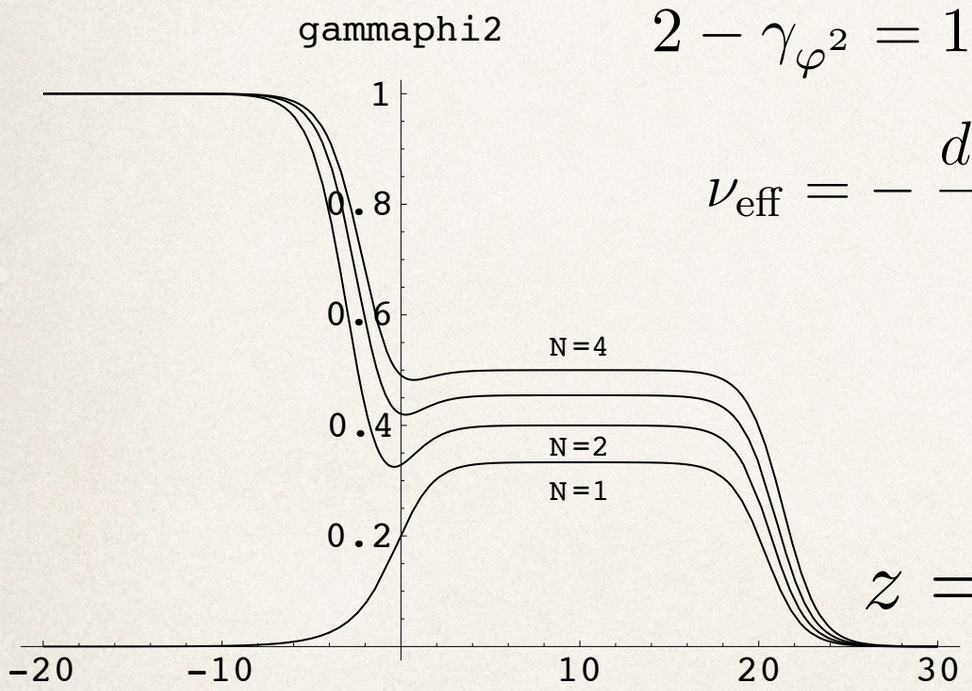
Not all renormalizations capture a crossover. Need a coarse graining/RG map that captures the changing nature of the effective degrees of freedom as a function of the "environment", e.g. as 3D effective degrees of freedom transform to 2D ones in a thin film

$$\chi = At^{-\gamma} f(Lt^{\nu})$$

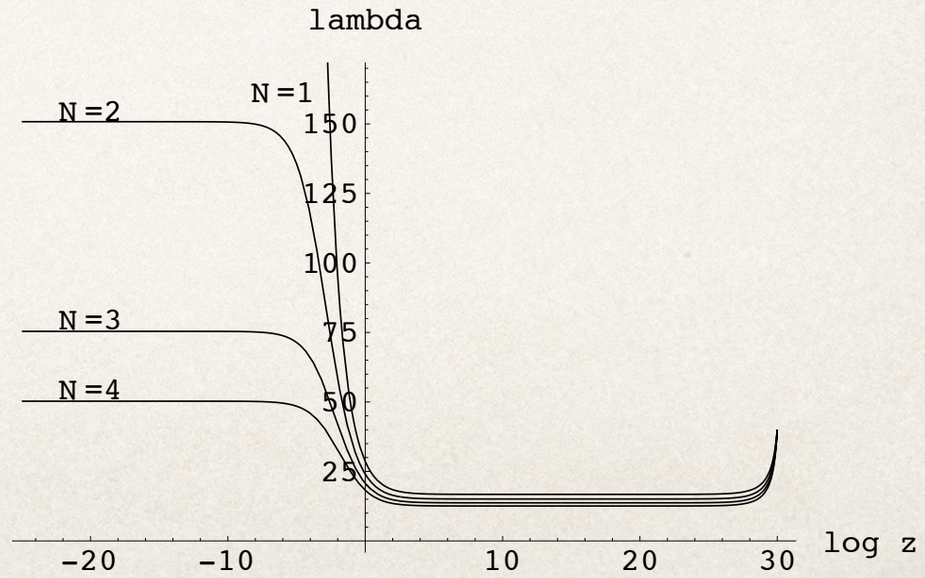
$2 - \gamma_{\varphi^2} = 1/\nu$ is a constant in the non-crossover case

$$\nu_{\text{eff}} = -\left.\frac{d\ln\xi(g)}{d\ln t}\right|_{H=0}$$

and hence γ_{φ^2} are functions in the crossover case - universal functions



$$z = L/\xi$$



The evidence so far…

1. Critical behaviour defied an appropriate quantitative description for many decades

2. The RG completely solved the problem giving an accurate quantitative description of the critical region and explaining scaling and universality as associated with fixed points of the RG

3. Standard criticality is “special” (not generic) in that it has to be tuned, i.e., is associated with a manifold in the space of parameters with relevant operators

4. Standard criticality is “boring” (phenomenologically poor) in that there are only two length scales involved - super-universality

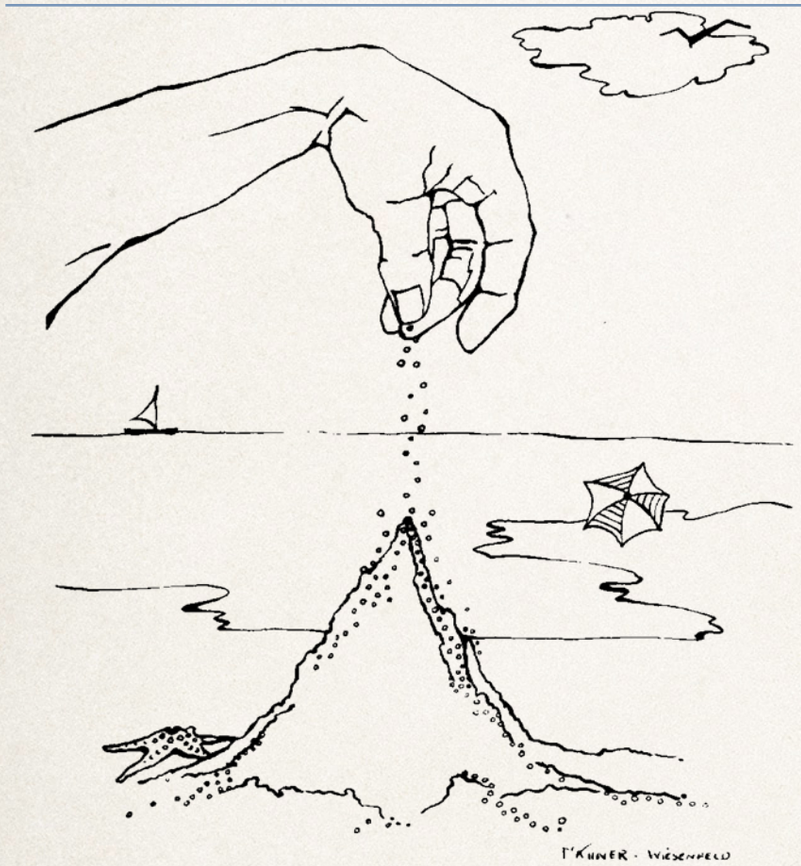
5. In real systems there are always other scales involved

6. The presence of other scales leads to crossover phenomena, with a richer phenomenology than standard critical phenomena

7. There are RGs that have been used to explain and describe many crossovers

Standard Criticality is "special" (not generic)

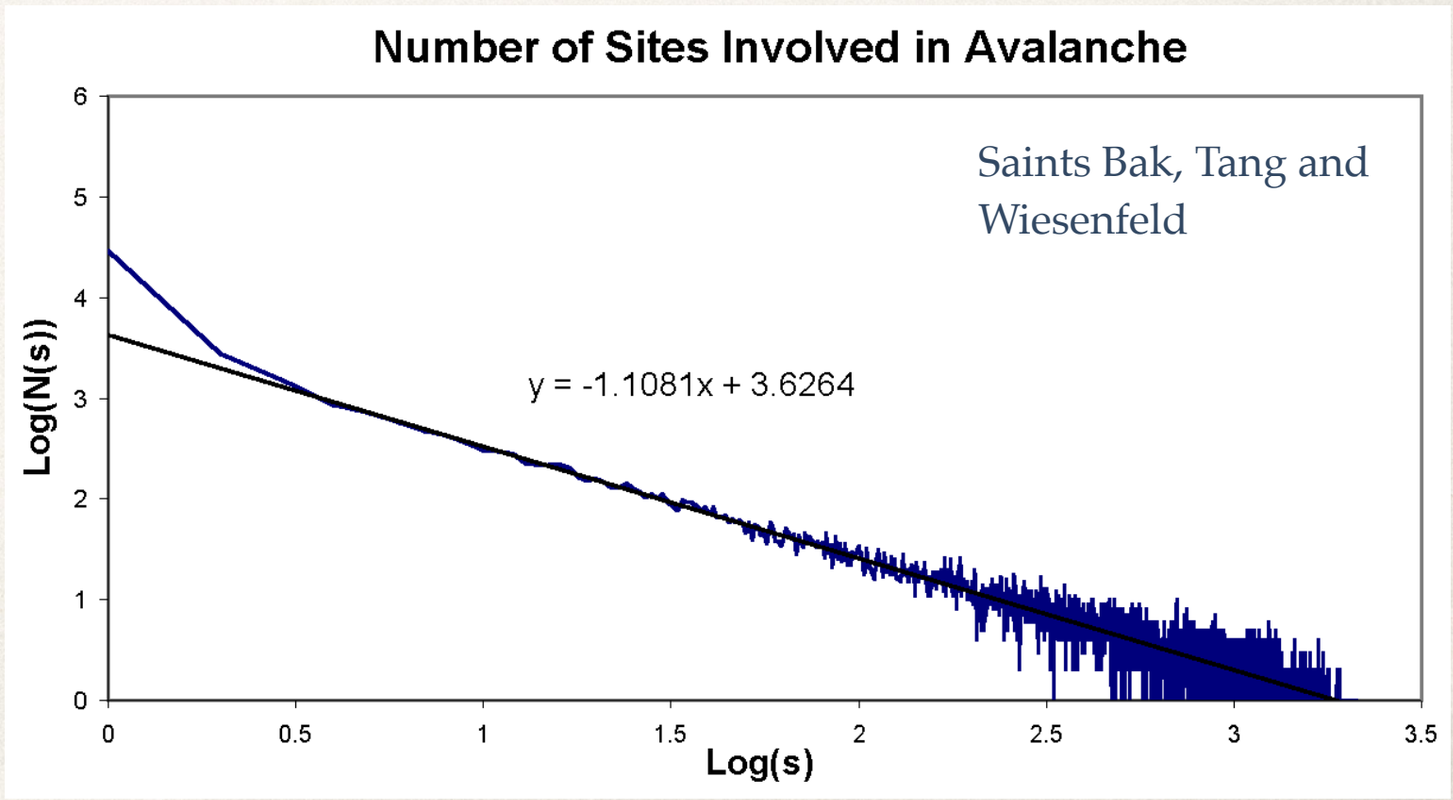
What about self-organised criticality? The Miracle of the Self-tuning



An explanation of everything and an explanation of nothing?

At least its a possible route out of the non-generic nature of criticality

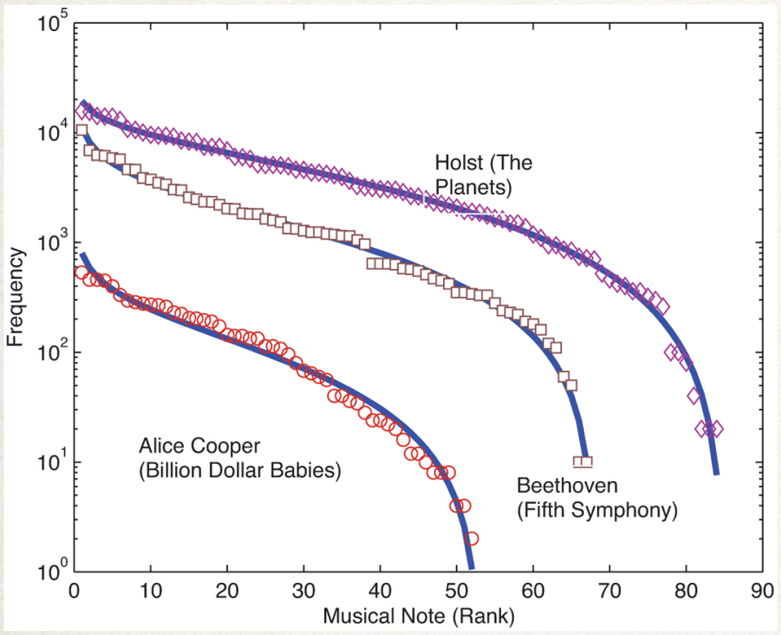
But it's still boring! There are still only one or two scales, the correlation length and a microscopic scale

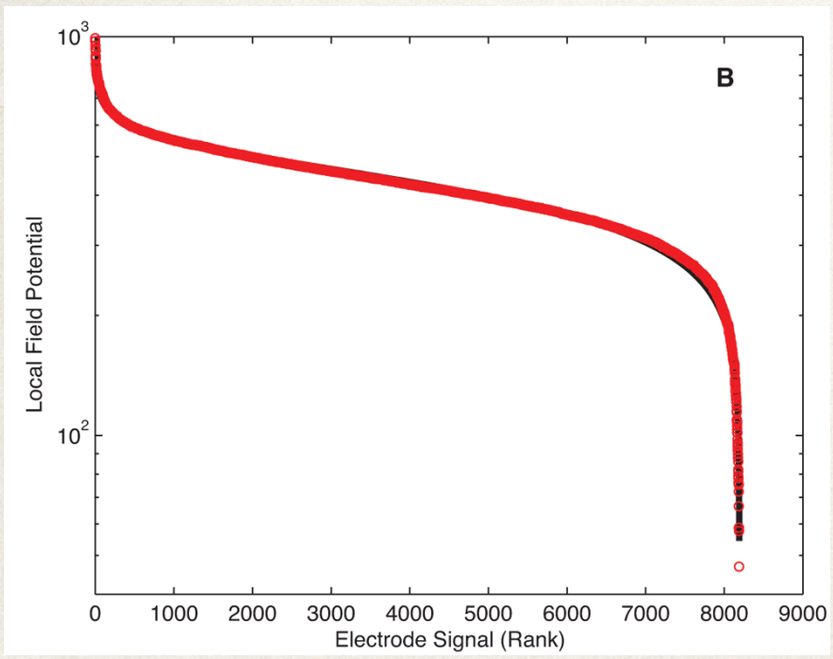


When are power laws really power laws?

Scaling behaviour in Complex Systems

The Miracle of the Beta Distribution

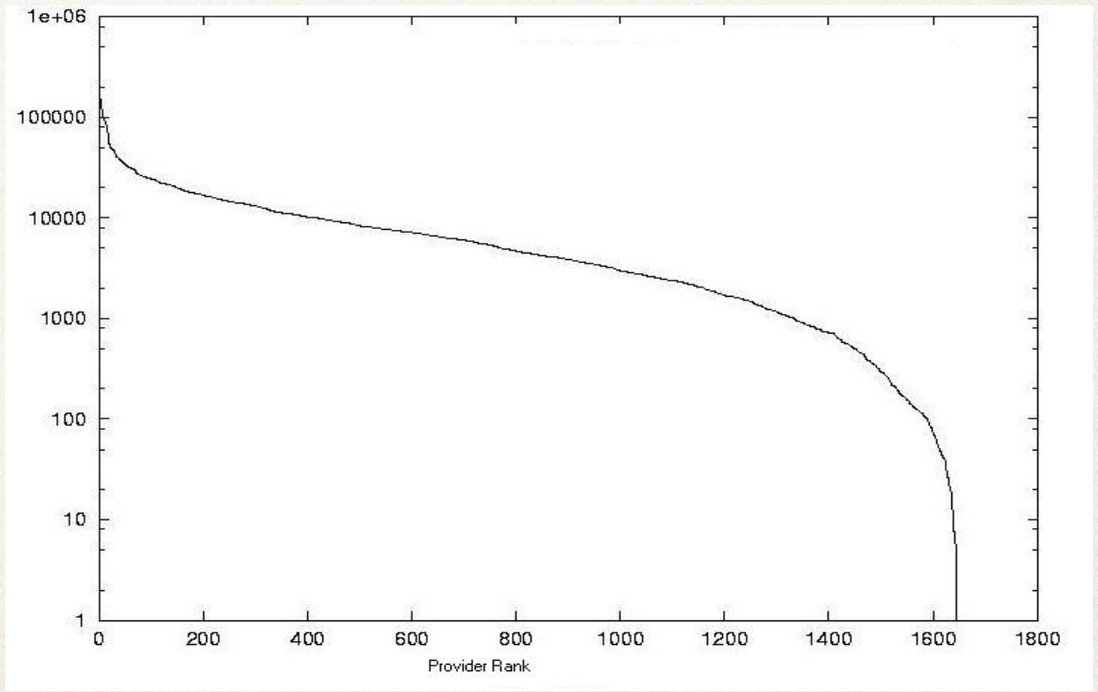




Martínez-Mekler G, Martínez RA, del Río MB, Mansilla R, Miramontes P, et al. (2009)
Universality of Rank-Ordering Distributions in the Arts and Sciences.
PLoS ONE 4(3): e4791. doi:10.1371/journal.pone.0004791

$$f(r) = A(N+1-r)^b / r^a$$

Is the phenomenology of Complex Systems more complex?



Graph of Medicaid payments versus rank of pharmacy provider for Medicare/Medicaid claims NJ (2002).

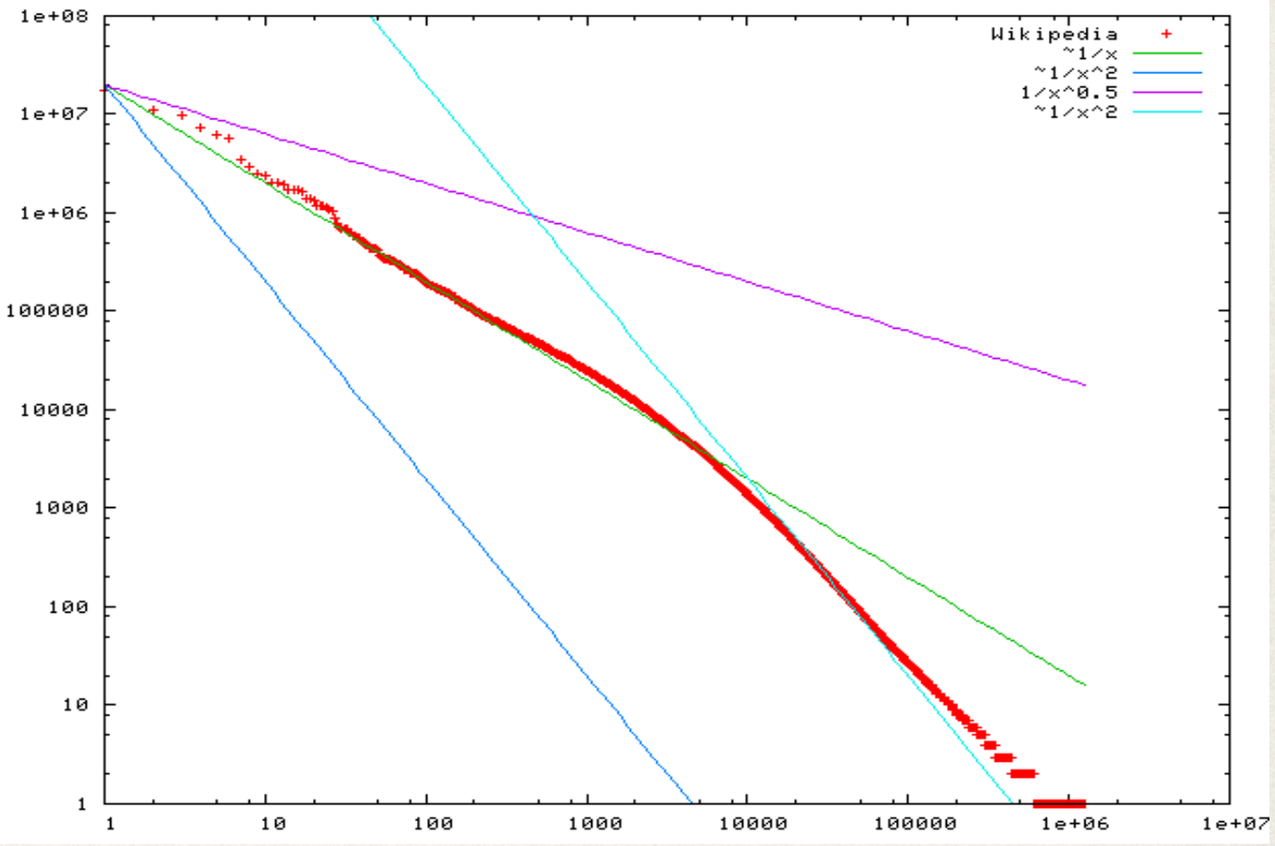
Stephens (2002)

Is Language on the "Edge of Chaos"? The Miracle of Saint Zipf

Zipf's law may be stated mathematically as:

$$f(k;s,N) = \frac{1/k^s}{\sum_{n=1}^{N} 1/n^s}$$

where N is the number of elements, k is their rank, and s is the exponent characterizing the distribution. In the example of the frequency of words in the English language, N is the number of words in the English language and, if we use the classic version of Zipf's law, the exponent s will be equal to unity. $f(k; s,N)$ will then be the fraction of the time the kth most common word occurs.



In general in natural language
s ~ 1.

So, language is on the "Edge of Chaos"!

Saint Zipf (Ulysees)

Does this give an adequate description of language? That we need only state that the frequency distribution of words is scale invariant with exponent s?

NO! So what's in language that's not in a ferromagnet or a sandpile?

The Miracle of the Complexity of Language

To be, or not to be--that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous fortune
Or to take arms against a sea of troubles
And by opposing end them. To die, to sleep--
No more--and by a sleep to say we end
The heartache, and the thousand natural shocks
That flesh is heir to. 'Tis a consummation
Devoutly to be wished. To die, to sleep--
To sleep--perchance to dream: ay, there's the rub,
For in that sleep of death what dreams may come
When we have shuffled off this mortal coil,
Must give us pause.

Human nucleotide sequence

AAAAGAAAAGGTTAGAAAGATGAGAGATGATAAAGGGTCCATTTGAGGTTAGGTAAT
ATGGTTTGGTATCCCTGTAGTTAAAAGTTTTTGTCTTATTTTAGAATACTGTGACTA
TTTCTTTAGTATTAATTTTTCCTTCTGTTTTCCTCATCTAGGGAACCCCAAGAGCAT
CCAATAGAAGCTGTGCAATTATGTAAAATTTTCAACTGTCTTCCTCAAAATAAAGAA
GTATGGTAATCTTTACCTGTATACAGTGCAGAGCCTTCTCAGAAGCACAGAATATTT
TTATATTTCCTTTATGTGAATTTTTAAGCTGCAAATCTGATGGCCTTAATTTCCTTT
TTGACACTGAAAGTTTTGTAAAAGAAATCATGTCCATACACTTTGTTGCAAGATGTG
AATTATTGACACTGAACTTAATAACTGTGTACTGTTCGGAAGGGGTTCCTCAAATTT
TTTGACTTTTTTTGTATGTGTGTTTTTTCTTTTTTTTTAAGTTCTTATGAGGAGGGA
GGGTAAATAAACCACTGTGCGTCTTGGTGTAATTTGAAGATTGCCCCATCTAGACTA
GCAATCTCTTCATTATTCTCTGCTATATATAAAACGGTGCTGTGAGGGAGGGGAAAA
GCATTTTTCAATATATTGAACTTTTGTACTGAATTTTTTTTGTAATAAGCAATCAAGG
TTATAATTTTTTTTAAAATAGAAATTTTGTAAGAAGGCAATATTAACCTAATCACCA
TGTAAGCACTCTGGATGATGGATTCCACAAAACTTGGTTTTATGGTTACTTCTTCTC
TTAGATTCTTAATTCATGAGGAGGGTGGGGGAGGGAGGTGGAGGGAGGGAAGGGTTT
CTCTATTAAAATGCATTCGTTGTGTTTTTTAAGATAGTGTAACTTGCTAAATTTCTT
ATGTGACATTAACAAATAAAAAAGCTCTTTTAATATTAGATAA

49

Something complex?

Something complex?

aaaa aaaa aaaa aaaa aaaa aaaa aaaa…

Something complex?

asmjgre fj sdjf s rege geoiie rgeasdffi…

Something complex?

1001 110 11001 1111 10101 1 10010 101 1101 1 10010 10010 …

Something complex?

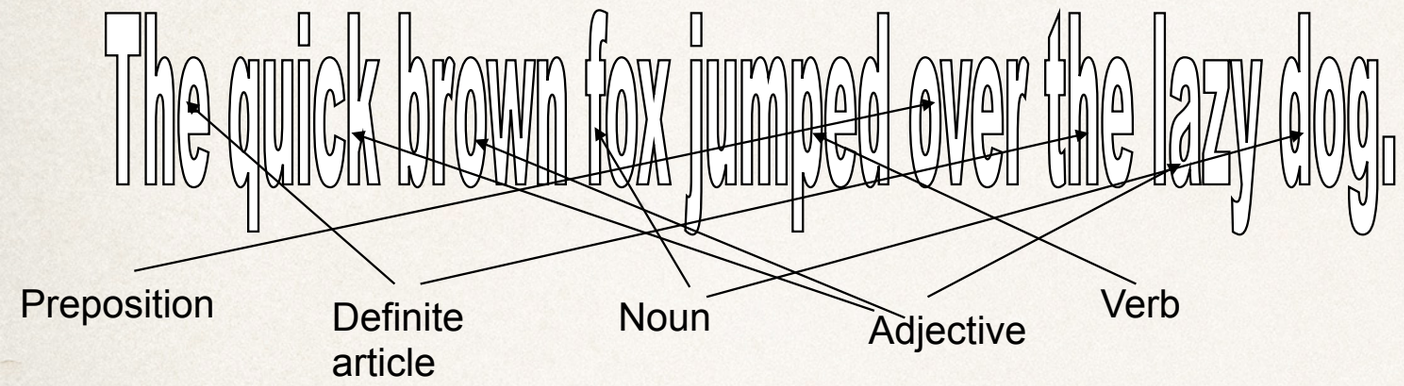
If you are married or are a man and woman living together as if you are married you must claim jointly …

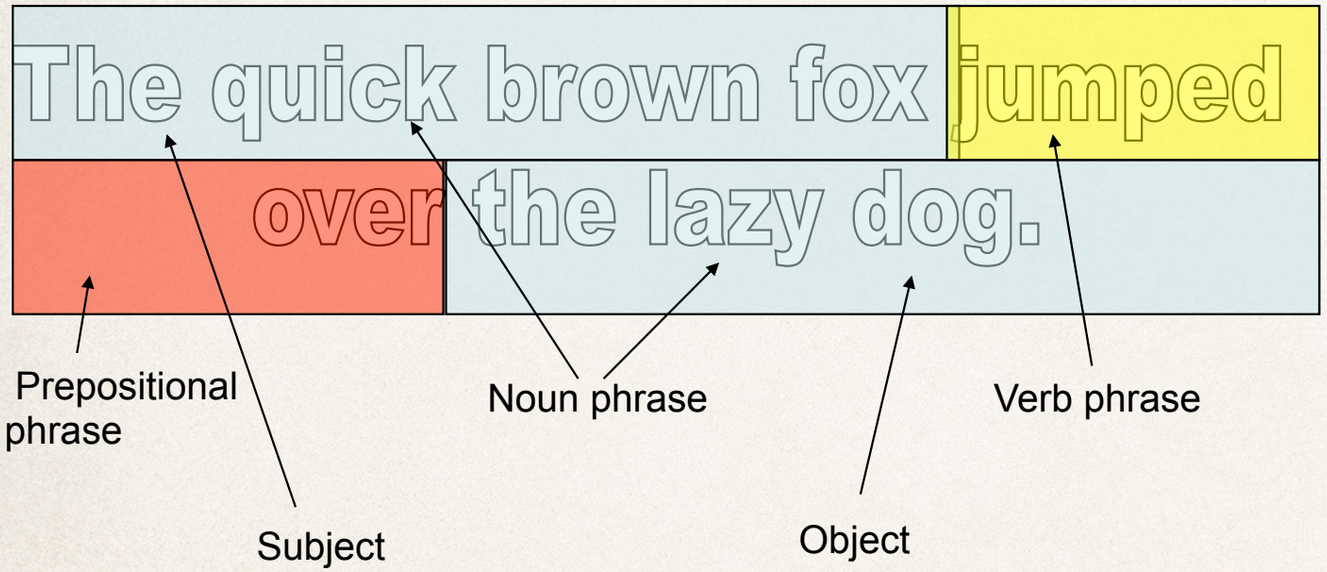
Language is full of scales

- Alphabet sizes aren't scale invariant
- Word sizes aren't scale invariant (average word size in English is about 5.67 letters/1.74 syllables). The longest is "pneumonoultramicroscopicsilicovolcanoconiosis" at 45 letters. So, there's less than two decades.
- Sentence sizes aren't scale invariant (average sentence size is about 20 words)
- Paragraph lengths aren't scale invariant (average size about 150 words)
- Book lengths aren't scale invariant (median book length is about 64,000 words)

In fact, its composed of Building Blocks

What's the fitness landscape of natural language?





The quick brown fox jumped over the lazy dog.

Gerald Cook is 86 this year.

Lexicographic

Very rugged landscape, few combinations of letters are valid (few combinations of nucleotides are genes) but smooth with respect to word content. No longer range interactions beyond the word.

Grammatical

Very rugged landscape, few orderings and combinations of words are valid. No longer range interactions beyond the sentence.

Semantic

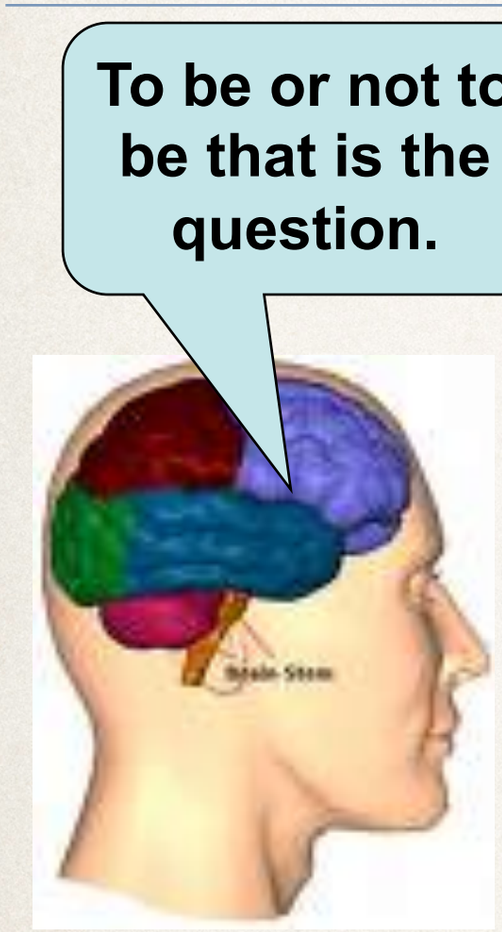
Very rugged landscape, few orderings and combinations of words are valid. Interactions at all scales!

The "micro" and the "macro" are linked

Complexity

Is Complexity Measurable?

The Complexity of Language



The measuring apparatus. Is it any good?

To be or not to be that is the question.

Para ser o no ser que es la pregunta.

Om te zijn of te zijn niet dat de vraag is.

あるためまたはないため質問である

Because of a certain or because it is not, it is question.

Because or it is not for the sake of, that having asked and being convinceu.

Being not to be for the sake of, or that that, you ask, are convinced.

It is that without having for the sake of, or, you ask, are convinced.

good

bad

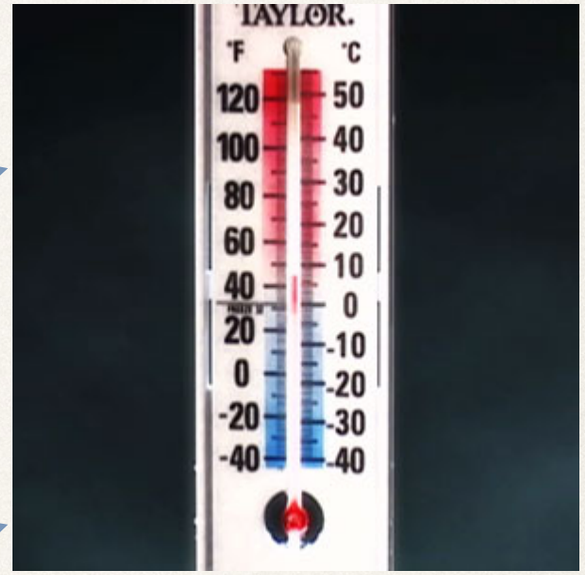
bad

good

Does this make the measurement of complexity subjective?

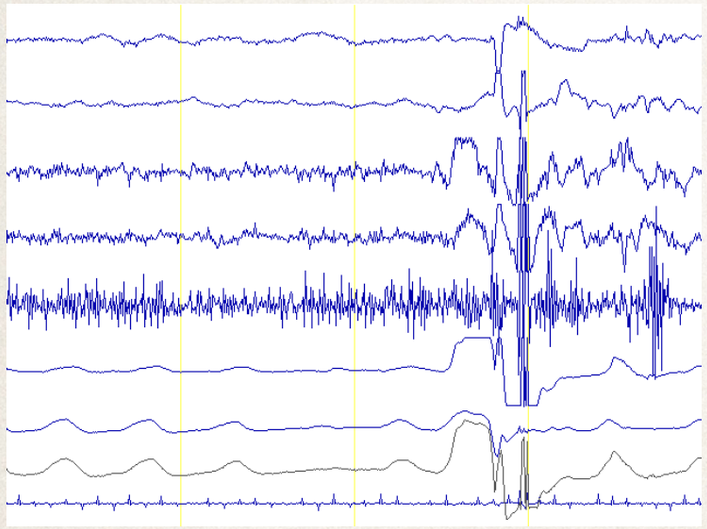
No, just the same thing happens in physics

A **good measuring apparatus needs to be calibrated**



good

bad



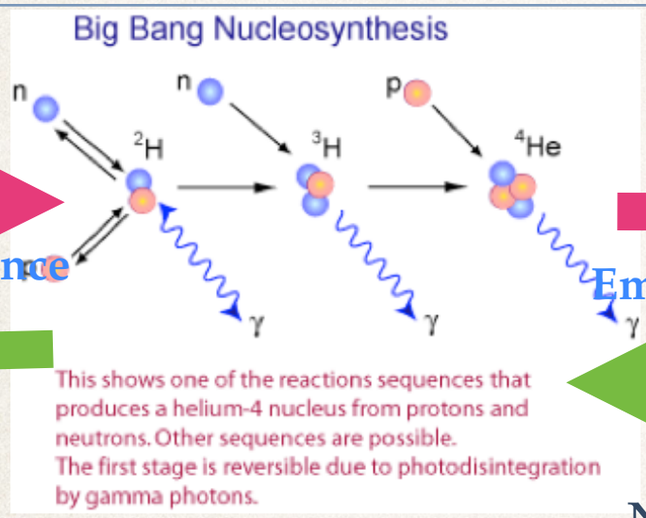
bad

good

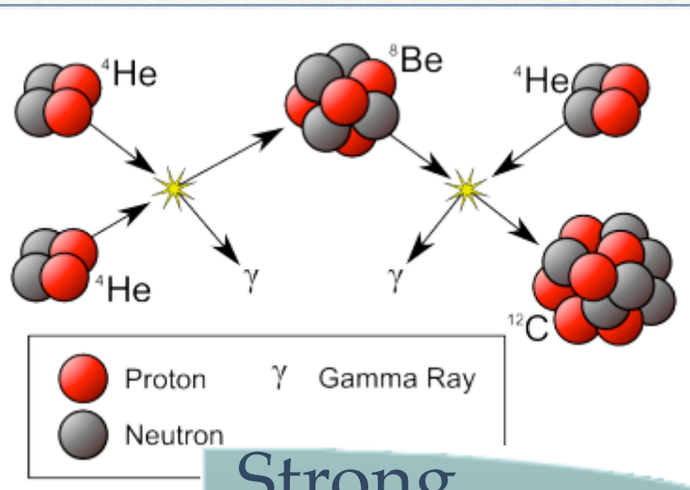
Complexity is a joint property of a system and its measuring apparatus

As is the physical world…

Emergence of structure and reductionism



Particle physics

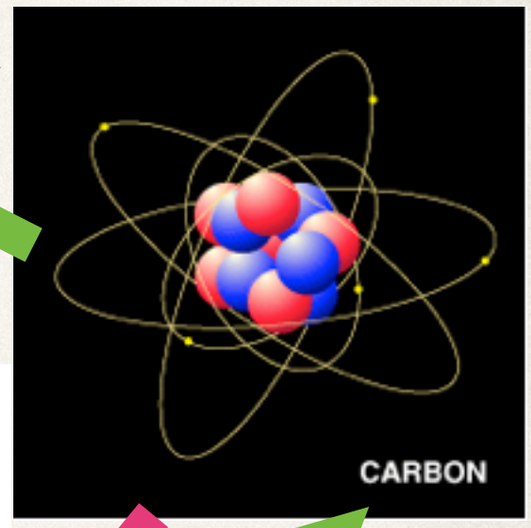


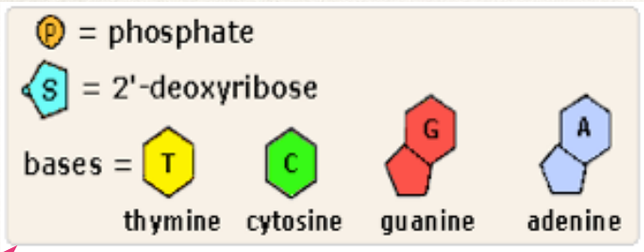
Nuclear physics

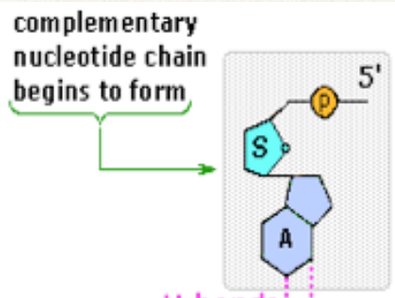
Strong interactions

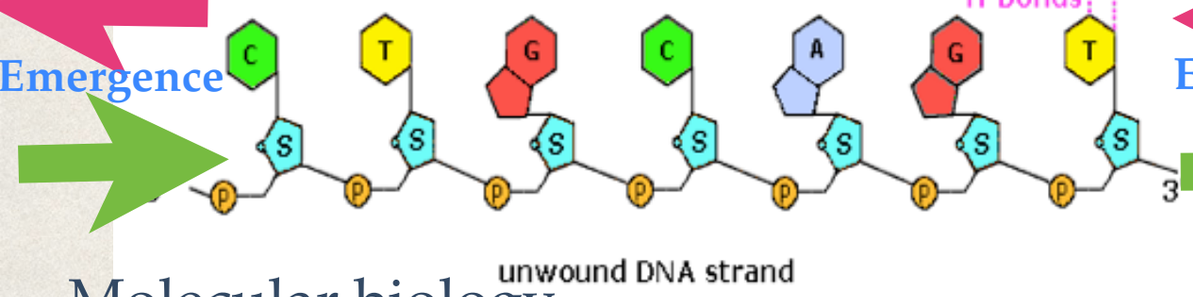
Weak

Atomic physics

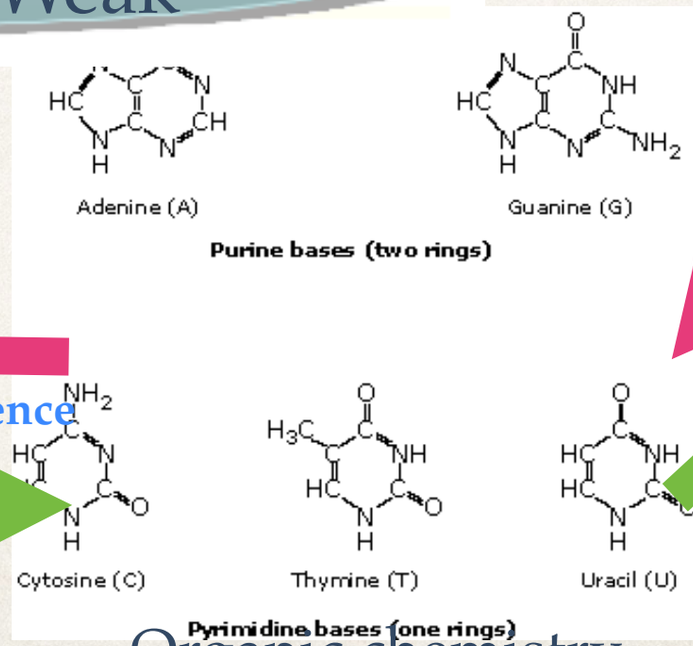








Molecular biology



Organic chemistry

Motter: Emergent networks - graphite and diamond

Emergence

Emergence

Emergence

Emergence

Emergence

Emergence

Everything here is a slave to the law

And the biological one…

Emergence of function

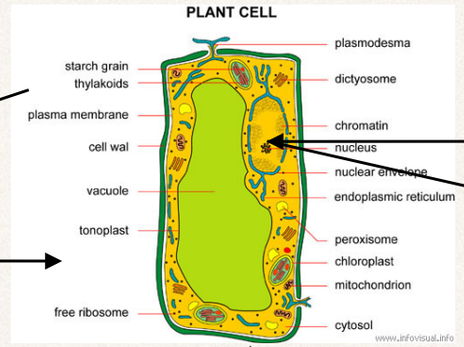
Botany, Ecology, Biology

10^{-1}m

Colour, size, form, leaves, roots, fruits, number of petals, number of cell types, number of genes, number of types of synthezised proteins …

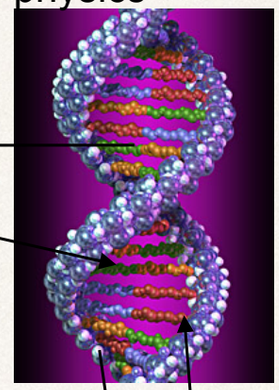
“Reductionism”

Cell Biology, Genetics



10^{-5}m

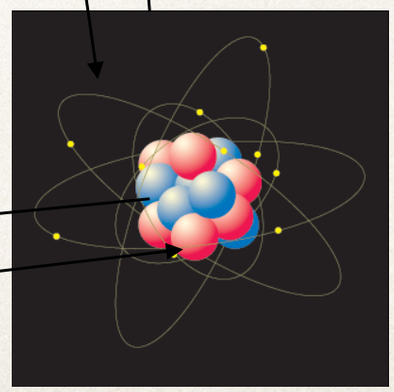
Biochemistry, biophysics, molecular physics



Macromolecule composed of 4 bases C, G, T y U
H,
C, N y O.

10^{-9}m

Chemistry, atomic physics, nuclear physics



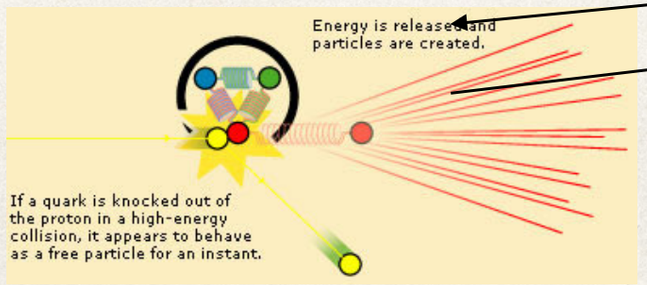
10^{-10}m

Number of electrons, number of protons and neutrons, their masses, charges

“Agentes”

Building Blocks

Física de partículas



Are physical and biological Building Block hierarchies the same?

No physicist loses sleep over the fact that we haven't been able to derive a many-body formulation of molecular physics from first principles atomic physics, much less from nuclear physics and electrodynamics, even less from elementary particle physics.

No physicist or chemist loses sleep that we haven't been able to describe the formation of a crystal of salt from the properties of sodium and chlorine atoms.

Biologists and doctors can and probably do lose sleep worrying how to reduce the incidence of HIV, or cancer, or diabetes, or…

What's the difference?

Effective theories work incredibly well in physics and chemistry.

Biological systems aren't machines!

They're Complex Adaptive Systems

More evidence…

1. The real word, both physical and biological, is not scale invariant, rather, what characterises it is a multitude of different scales, each associated with a particular effective degree of freedom - Building Blocks - that form a hierarchy.

2. The associated phenomenology is immensely rich.

3. We have no quantitative, theoretical framework to describe such hierarchies (the problem of crossovers x 1000! Is the RG our best hope?) Successful mathematical modelling has been with effective theories associated with “one” scale.

4. The world is so full of scales that we think that criticality is the, rather ubiquitous, exception that needs to be explained. However, we can also think that the natural state of the world is scale invariant and then we must explain why there exist so many scales and perhaps every, very ubiquitous, scale requires an explanation.

5. The only way of building complexity is through Building Blocks - the problem of search.

6. The Building Blocks in biological systems have functions.

From the science of yesterday to the science of tomorrow…

the challenge of Complex Adaptive Systems

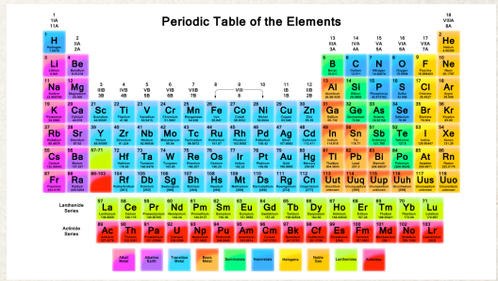
How we do science in a nutshell… The Faith

- **The Scientific method:** Systematic observation, measurement, and experiment, and the formulation, testing, and modification of hypotheses

- **Phenomenology** - a body of knowledge that relates empirical observations of phenomena to each other, in a way that is *consistent* with fundamental theory, but is not directly derived from theory.

- **Taxonomy** - the practice and science of classification. A classification of things or concepts, as well as to the principles underlying such a classification.

  - Examples: Medicine, astronomy, chemistry, biology, physics,...

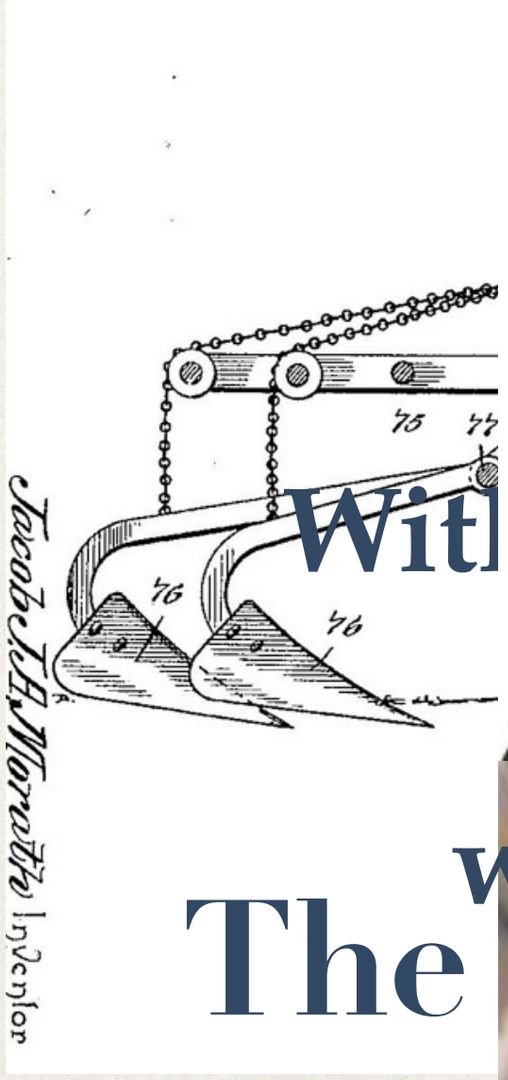


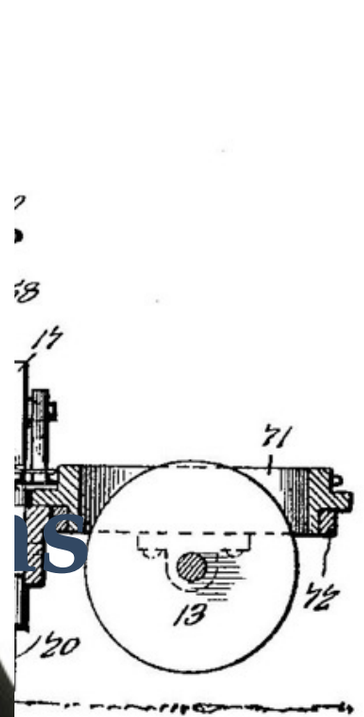
- **Scientific law** - when a particular phenomenon always occurs if certain conditions are present

The worldview of the last 3 centuries: The Doctrine

In fact...

How es?





With s

we are slaves of the law

The ne

Universality

We're all equal under the law

But in physics and chemistry, once you've seen one perfect gas there's really not a lot to say In general, you don't need to know At any interesting place that much data

In Complex Adaptive Systems however...

t you

Imagine what you can say about a city

versus

a crystal as big as a city!

You can say a lot about a Complex Adaptive System and each thing you say depends on a lot of other things

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Any observable of a complex system depends on a whole host of other factors

P(A,B,C,D,...; t | a,b,c,d,...; t′)

Diabetes Renal failure Obesity Father had diabetes

Angina Leukaemia SNP Rs7903146 45 mins exercise per week

Many effects Many causes

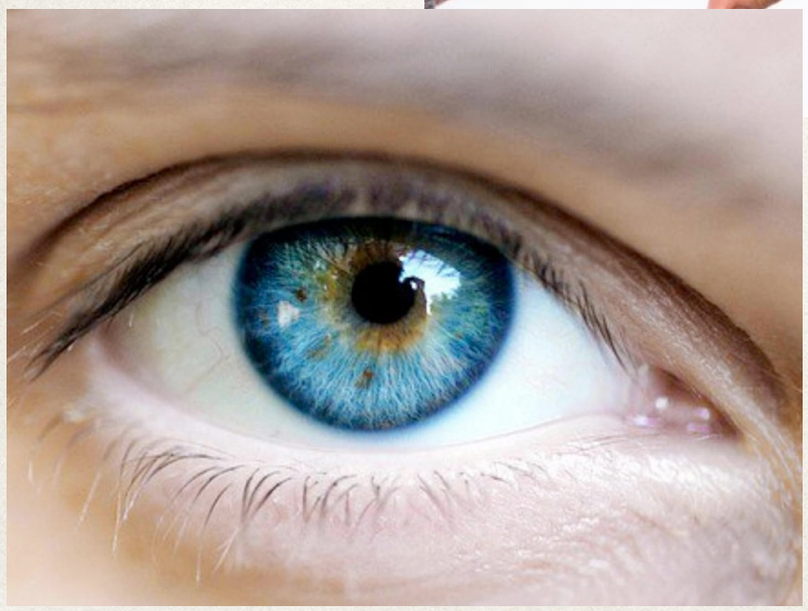
From the “micro” to the “macro”

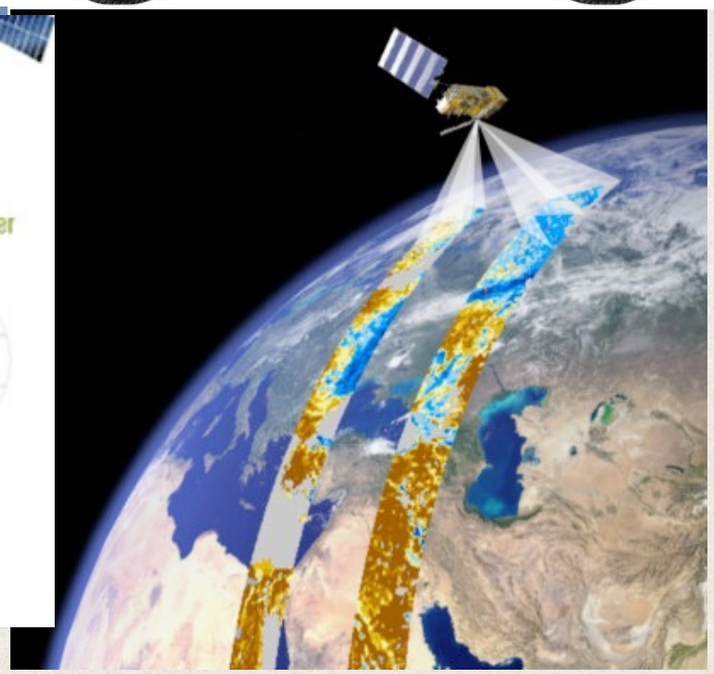
Many disciplines

The Miracle of the Data

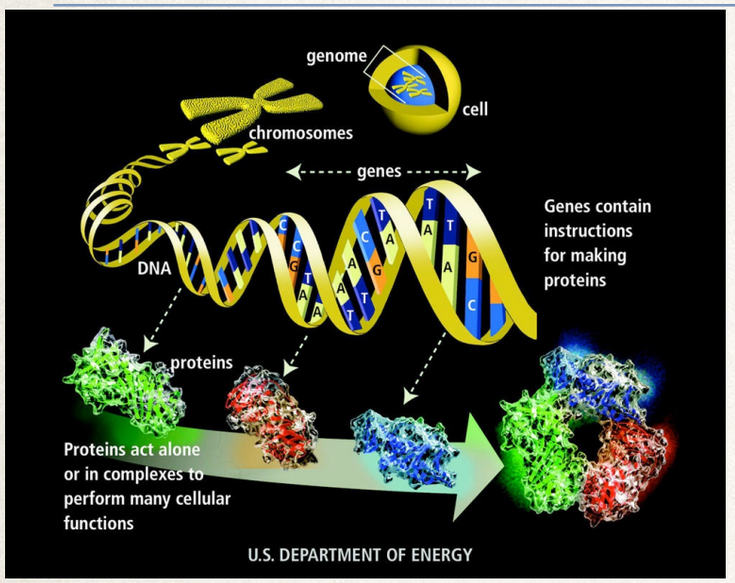
Fortunately....
You need a lot of data to describe complexity
we are in the middle of a revolution

A revolution in data gathering...

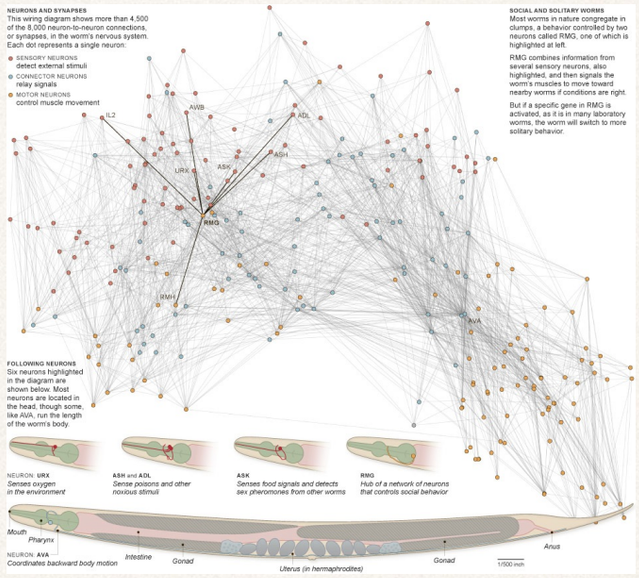




A revolution in data storage...



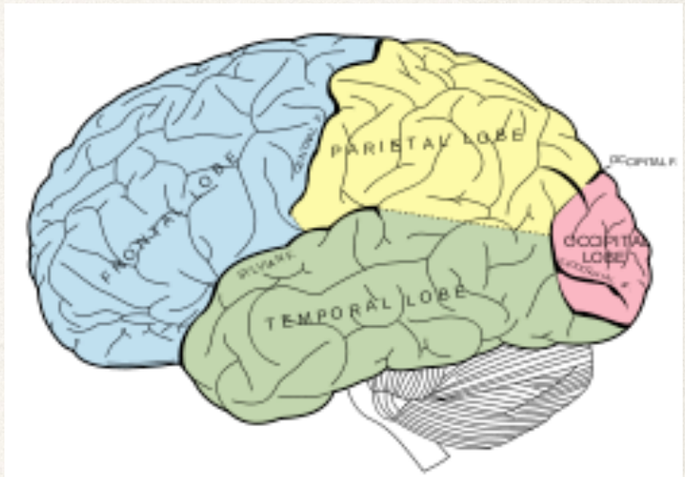
Genomes 1kB - 1.5 GB



Worm neural network 0.3MB

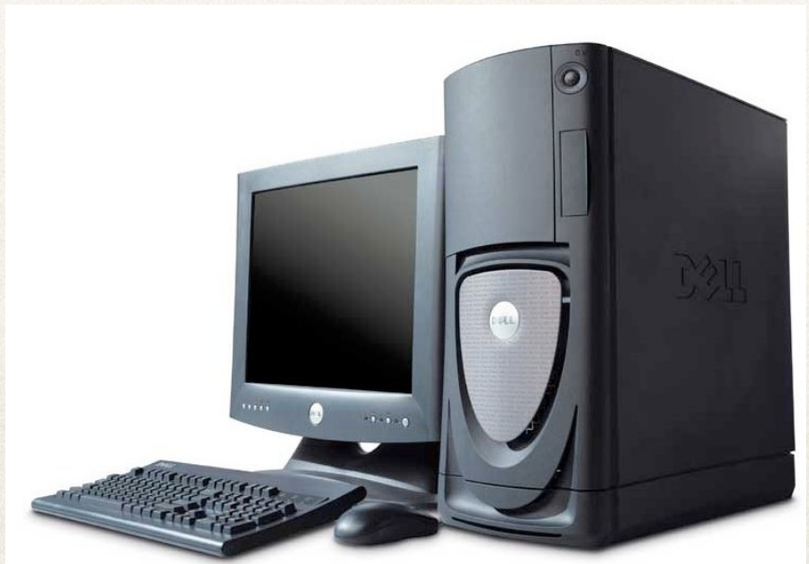
Human brain

10-100 Terrabytes



In electronic form 1 zettabyte

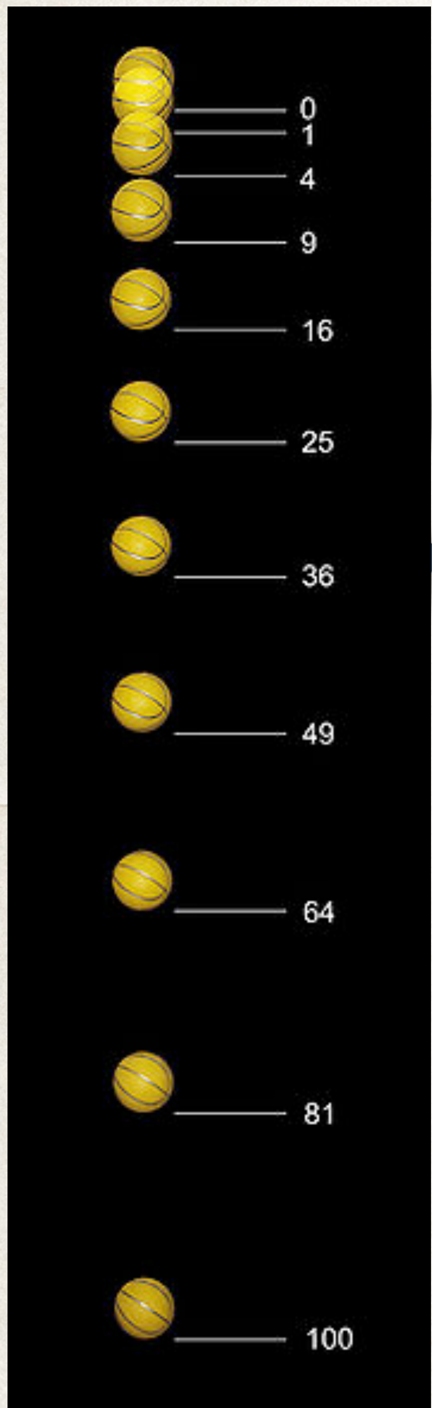
Raw data is processed and stored



All the books in the world

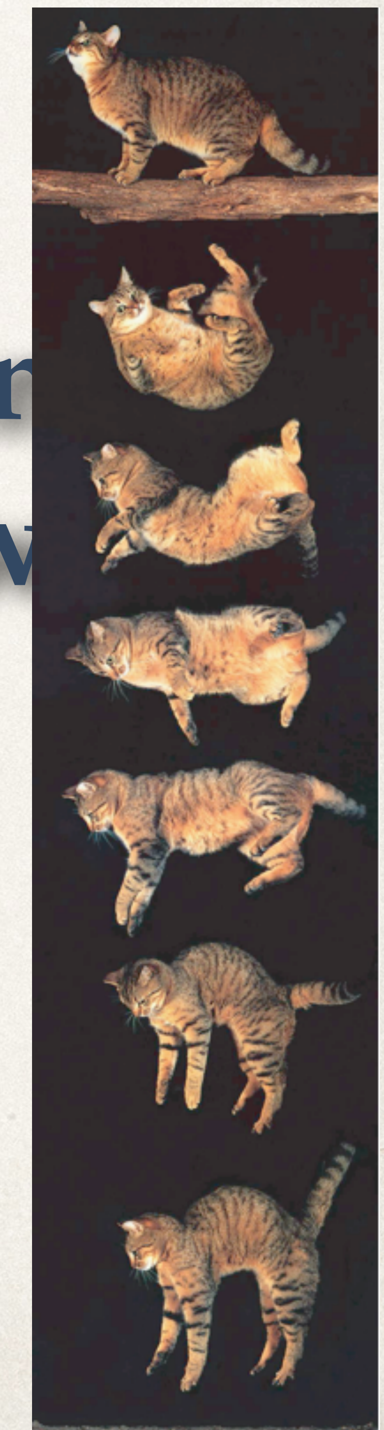
30-50 Terrabytes

Adaptation and the freedom to choose



Mechanistic

Adaptive



The cat obeys the same laws of physics as the ball.

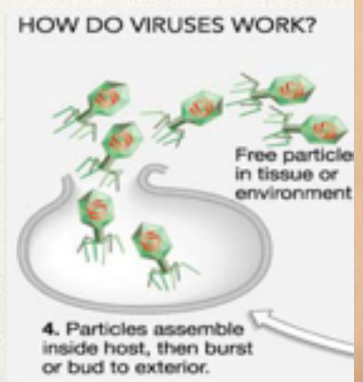
But it's not a "slave" to them.

e difference between complex an
ple systems is the difference betw
"being" and "doing".

The *evolution* of function is the revolution that allowed systems to escape the tyranny of the laws of physics.

Complexity is a consequence of that revolution.

Adaptation and Decision Making

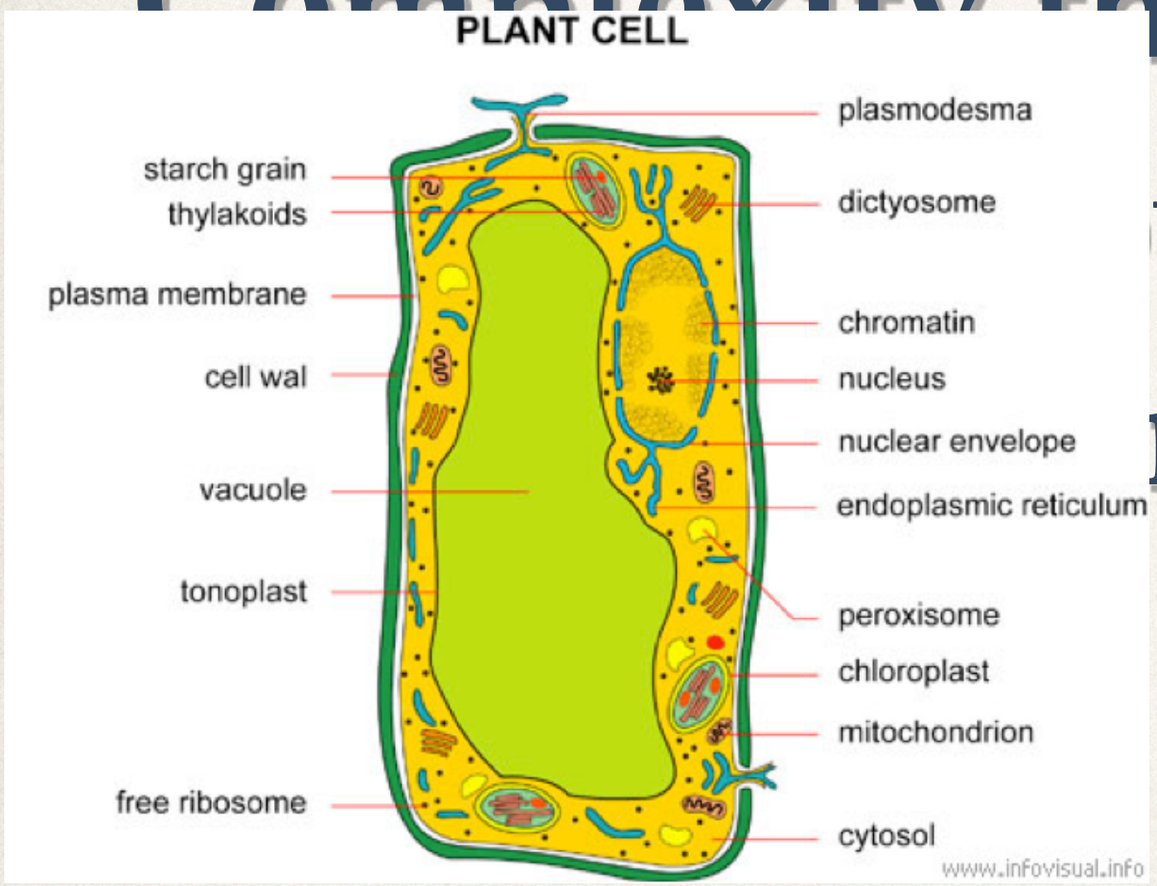


There are good decisions
and there are bad decisions

Building Blocks, Multi-tasking and Specialisation

The advantages of specialisation and the disadvantages

Complexity through evolution



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Modeling complexity

To make a mathematical model of a dynamical system...

we need a space of states and update rules that tell us how to get from one state to another

Modeling complexity

From the simple to the complex and from the complex to the simple

This isn't describing complexity

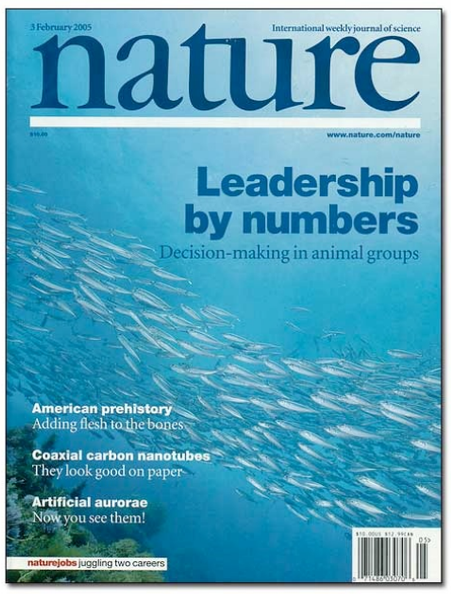
$$\mathbf{d}_i(t+\Delta t) = \sum_{j \neq i} \frac{\mathbf{c}_j(t) - \mathbf{c}_i(t)}{|(\mathbf{c}_j(t) - \mathbf{c}_i(t))|} + \sum_{j=1} \frac{\mathbf{v}_j(t)}{|\mathbf{v}_j(t)|}$$

Competition between short-range repulsion and longer range atraction between "particles"

$$\hat{\mathbf{d}}_i(t+\Delta t) = \mathbf{d}_i(t+\Delta t)/|\mathbf{d}_i(t+\Delta t)|$$

$$\mathbf{d}_i{}'(t+\Delta t) = \frac{\mathbf{d}_i(t+\Delta t) + \omega \mathbf{g}_i}{|\hat{\mathbf{d}}_i(t+\Delta t) + \omega \mathbf{g}_i|}$$

Equation for "charged" particles following an external force g_i



Couzin, I.D. , Krause, J., Franks, N.R. & Levin, S.A. (2005) Nature, 433 , 513-516.

Evolved Virtual Creatures

Here we see the
emergent phenomenon
with function

We begin with "building blocks"
and we end up with creatures
by the laws of physics (mechanics)

Examples from
work in progress

The Paradox of Complexity

We experience and manage complexity every day,...
unconsciously
"Consciously", we look always to single, simple causes...

capitalism, communism, rich, poor, PRI, PRD,
junk food, soft drinks, colonialism,...

Why?

Evolution has created
complex adaptive systems
Have we just traded
Natural selection also enslaves
by one master for another?
How far does free will go?
the
tyranny of the laws of physics

Conclusions

- Criticality has played a miraculous role in statistical mechanics and condensed matter theory leading to huge conceptual and theoretical advances - principally the RG and an understanding of the origin of scaling and universality.

- It appears in many different contexts in the physical, biological and social sciences.

- Power law scaling exists because there is no characteristic scale - whether it happens a surprising/unsurprising amount is somewhat subjective - like “emergence” itself.

- Maybe the puzzle is better seen by asking not why there are so many systems without a scale (statistical physics) but why there are so many systems with a scale (particle physics).

- Real systems always have multiple length scales - crossovers.

Conclusions

- The phenomenology of Complex (Adaptive) Systems is immensely richer than that of physical systems and especially that of critical systems.

- To describe the rich phenomenology of Complex (Adaptive) Systems requires a huge amount of data and therefore has required a data revolution almost all of the data of which is “non-scientific”.

- There’s much more data about Complex (Adaptive) Systems than there is capacity for humans (and even less scientists) to study it. New modelling paradigms such as data mining are required. Data mining is not only the best to attack this data, its also the appropriate way to develop a better phenomenological and taxonomic understanding of Complex (Adaptive) systems

Conclusions

- This rich phenomenology of Complex (Adaptive) Systems comes about from the existence of a hierarchy of important scales.

- These scales are associated with Building Blocks which, in the case of adaptive systems, have function.

- Building Blocks are the only way by which Complex (Adaptive) Systems and complexity may emerge.

- The difference between physics/chemistry and biology/social sciences is the difference between “being” and “doing” - Building Blocks in biology/social sciences have function and specialisation.

- The concepts and tools of present day science, and especially physics, are not adequate for describing Complex Adaptive Systems and complexity.

Saint Criticality or Saint Building Block…
You decide.

As we don't have adequate conceptual or theoretical frameworks in which to understand Complex (Adaptive) Systems or complexity, here are some homework problems…

How do we model adaptation?

How do we classify complex systems?

How do we obtain and integrate data?

How do we model the emergence of function?

How do we model the emergence of multi-tasking and specialisation?

How do we mathematically model macro-evolution?

How do we mathematically model meta-evolution?