What is Complexity?

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“Definitions”

- in computer science…?

- **Computational complexity** – refers to the computational resources necessary to solve a given problem
- **Descriptive complexity -** of a string is the length of the string's shortest description in some description language

In both manifestations the most "complex" problems are random!

Some other “definitions”…

- A complex system is a highly structured system, which shows structure with variations (Goldenfeld and Kadanoff)
- A complex system is one whose evolution is very sensitive to initial conditions or to small perturbations, one in which the number of independent interacting components is large, or one in which there are multiple pathways by which the system can evolve (Whitesides and Ismagilov)
- A complex system is one that by design or function or both is difficult to understand and verify (Weng, Bhalla and Iyengar)
- A complex system is one in which there are multiple interactions between many different components (D. Rind)
- Complex systems are systems in process that constantly evolve and unfold over time (W. Brian Arthur)
- “Complex things exhibit complex behavior” (Parisi)

You always see: “many degrees of freedom” and “non-linear” – that just about covers everything! Even quantum field theory, where we have an infinite number of degrees of freedom.

Effective Complexity (Gell-Mann)

- Descriptive Complexity, but measured not on an observed phenomena but through a subjective interpretation of interest to the observer, i.e., a *model* - an algorithm for specifying a probability distribution over the observed data.

But this sounds very subjective, as it depends on our model, how good it is and how well we can test it. Also, what data?

Phenomenology and taxonomy

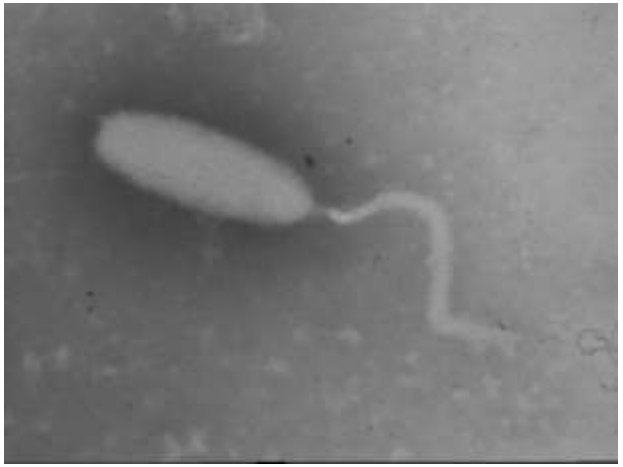
“Physical” Complexity

So what do we know for sure **is** complex?

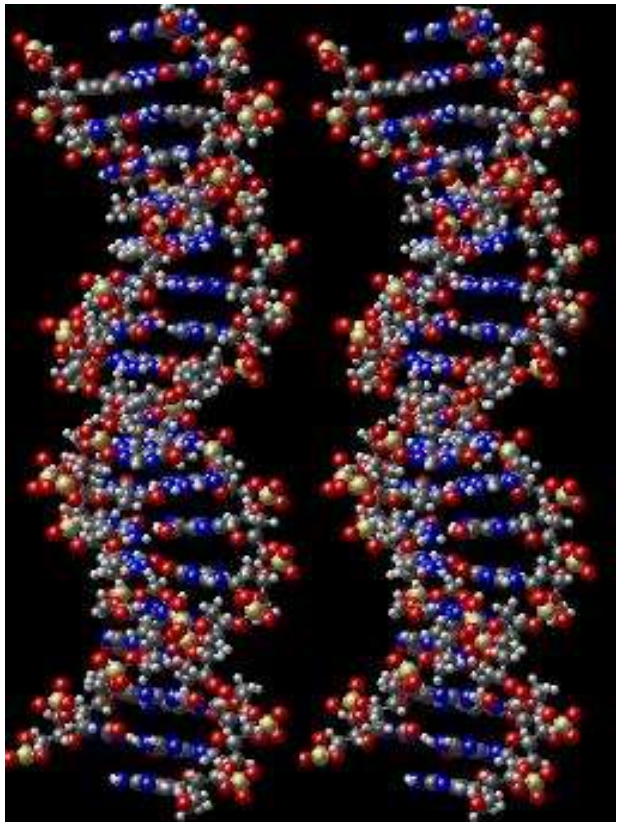
Well, what about this?

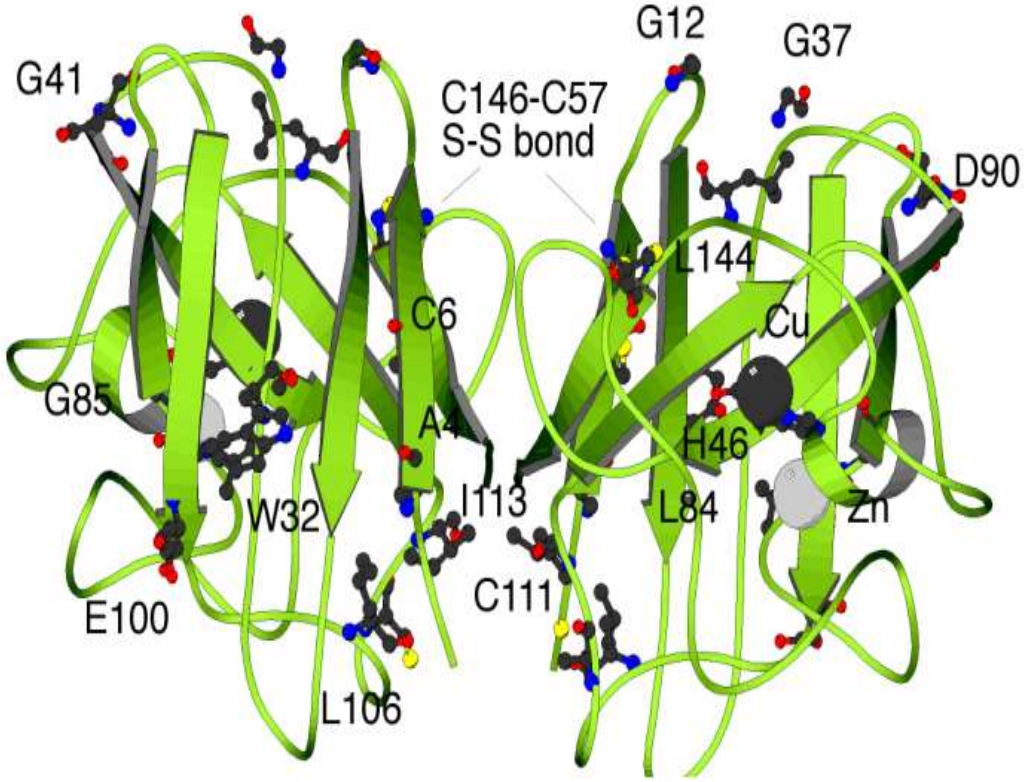
No?…this then…?

And this …?



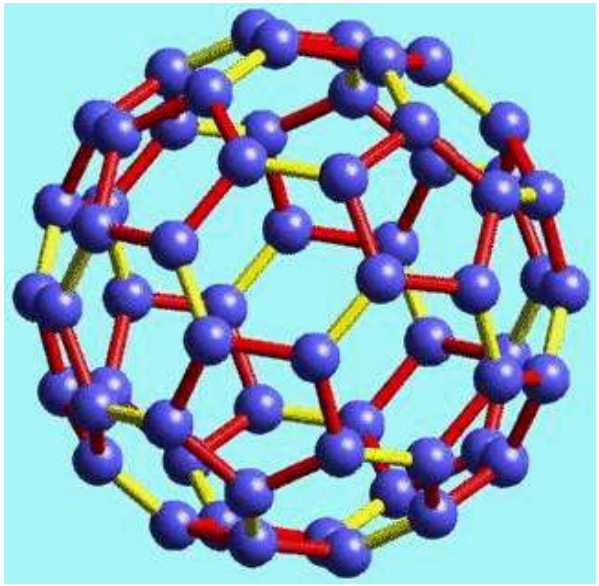
And these?



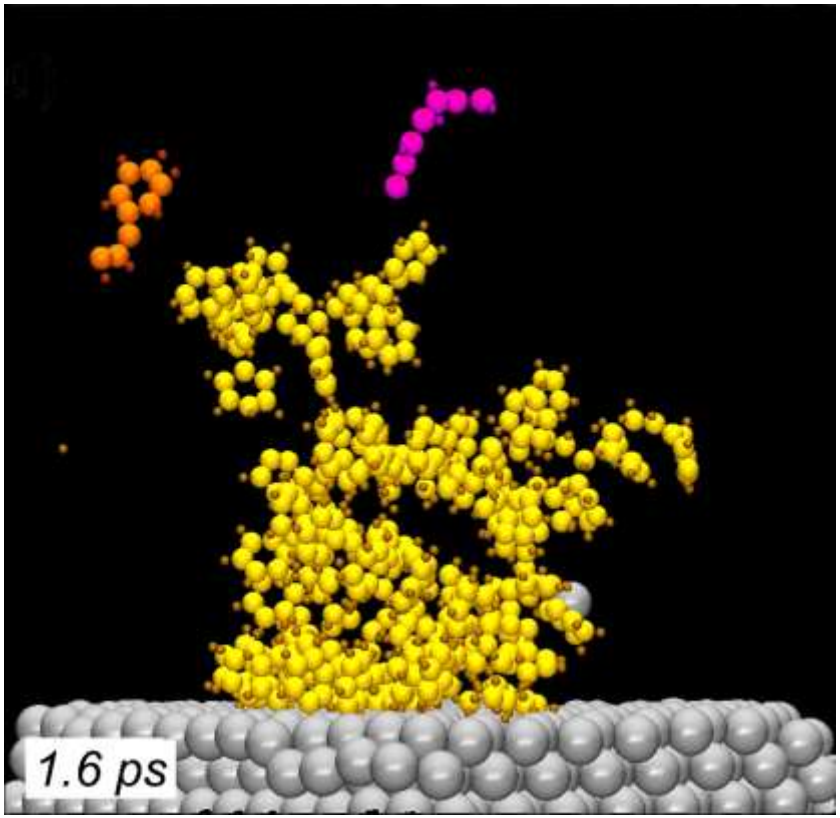


Model illustrating the formation of a misfolded species (M) from a folding intermediate (I). The region of the protein that misfolds is shown in red. The misfolded protein itself, or a self-assembled form, may be toxic to cells, leading to disease. The black arrows represent the relative rates of the various conformational events under native physiological conditions in the absence of mutation. The blue dash arrows represent the possible effects of mutation.

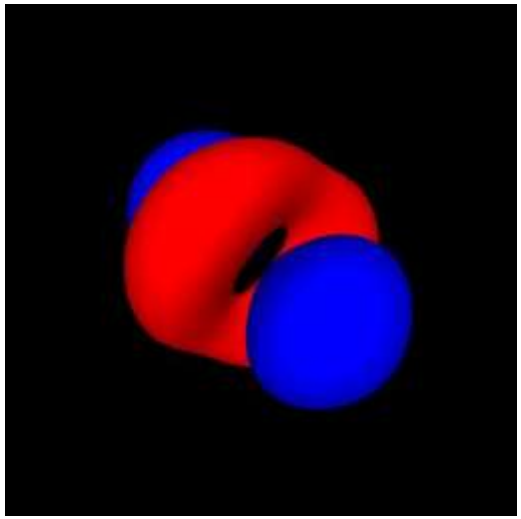
And what about these?



Buckyball C_{60}



Polystyrene on a silver surface



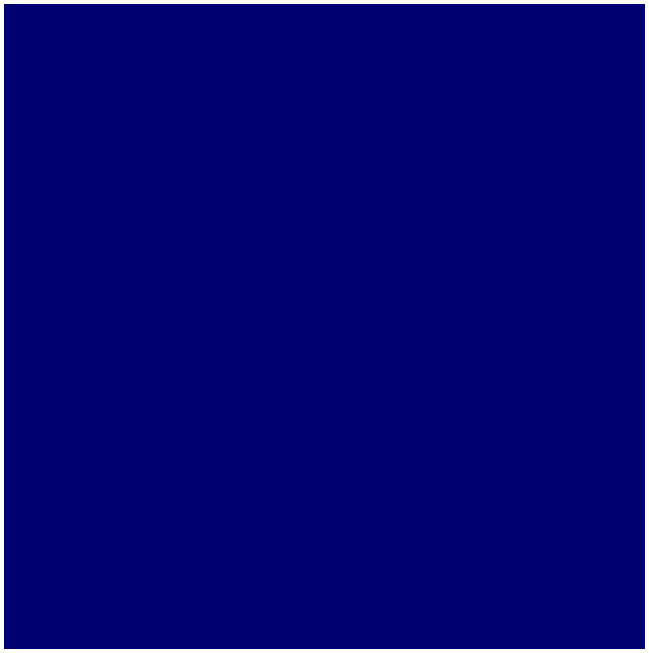
n=3, l=2 energy level of H

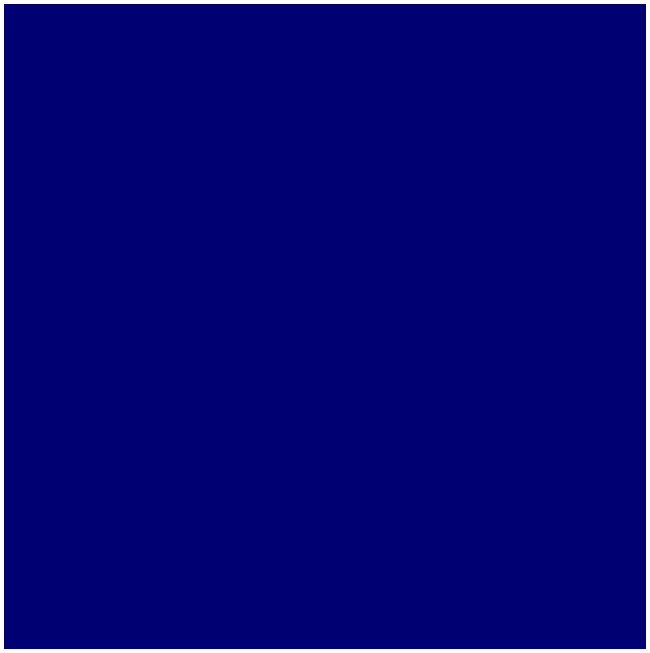
So, maybe we can agree on what is definitely complex, and what is definitely not complex. But where do we change from one to the other?

On the “Edge of Chaos” in “micro”-physics?

Barkhausen effect – “avalanches” of magnetic domains

“Dirty”





Near critical

Typical critical phenomenon showing <u>collective behavior</u> and scaling $Y \sim X^a$

But…

- Only one important length scale – the correlation length – that governs the scale of “collectivity”; Scale invariant near critical point (phase transition) – maximal “collectivity”
- Only one type of effective degree of freedom – a magnetic domain “avalanche”, but …
- Complex? Once the spectrum of “avalanche” sizes is given then there’s nothing much more to be said. Not very interesting living on the “Edge” in physics!
- The same is true for other canonical critical or self-organised critical phenomena

So what does distinguish the phenomena that we "agree" are complex from those that we "agree" aren't complex?

- A "hierarchy" of many different length scales
- Effective degrees of freedom ("collectivity") at different length scales are qualitatively different with different effective interactions

- Systems are adaptive
- Dynamical evolution depends on many different rules/strategies
- Systems "learn" (feedback from environment to system which is then used to update rules)

- More complex "behavior" (the "phenotype")

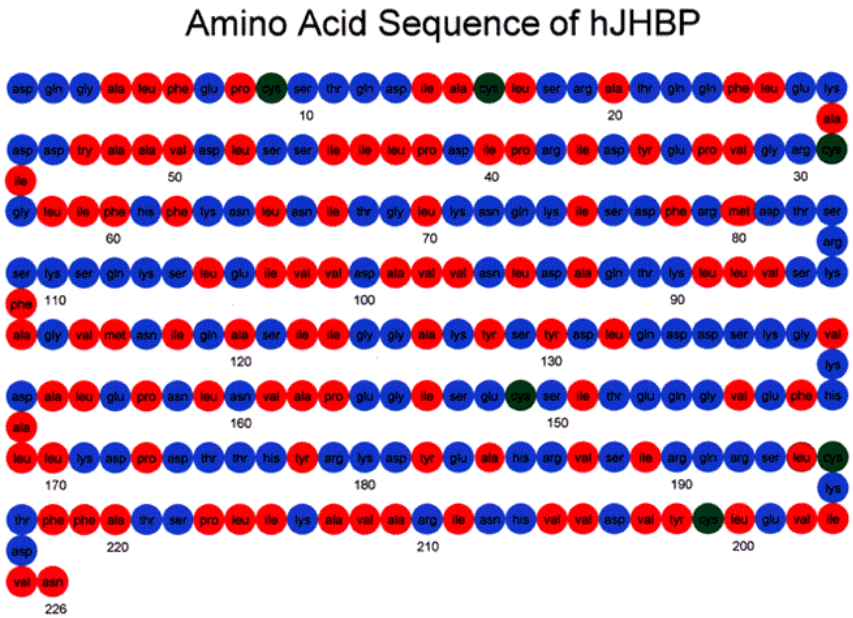
The trouble is that the definitions of complexity given before do not discriminate – too many false positives!

Perhaps contain some necessary conditions but certainly not sufficient

“Symbolic” Complexity

To be, or not to be--that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous fortune
Or to take arms against a sea of troubles
And by opposing end them. To die, to sleep--
No more--and by a sleep to say we end
The heartache, and the thousand natural shocks
That flesh is heir to. 'Tis a consummation
Devoutly to be wished. To die, to sleep--
To sleep--perchance to dream: ay, there's the rub,
For in that sleep of death what dreams may come
When we have shuffled off this mortal coil,
Must give us pause.

What about complexity in this case?



Human nucleotide sequence

AAAAGAAAAGGTTAGAAAGATGAGAGATGATAAAGGGTCCATTTGAGGTTAGGTAAT
ATGGTTTGGTATCCCTGTAGTTAAAAGTTTTTGTCTTATTTTAGAATACTGTGACTA
TTTCTTTAGTATTAATTTTTCCTTCTGTTTTCCTCATCTAGGGAACCCCAAGAGCAT
CCAATAGAAGCTGTGCAATTATGTAAAATTTTCAACTGTCTTCCTCAAAATAAAGAA
GTATGGTAATCTTTACCTGTATACAGTGCAGAGCCTTCTCAGAAGCACAGAATATTT
TTATATTTCCTTTATGTGAATTTTTAAGCTGCAAATCTGATGGCCTTAATTTCCTTT
TTGACACTGAAAGTTTTGTAAAAGAAATCATGTCCATACACTTTGTTGCAAGATGTG
AATTATTGACACTGAACTTAATAACTGTGTACTGTTCGGAAGGGGTTCCTCAAATTT
TTTGACTTTTTTTGTATGTGTGTTTTTTCTTTTTTTTTAAGTTCTTATGAGGAGGGA
GGGTAAATAAACCACTGTGCGTCTTGGTGTAATTTGAAGATTGCCCCATCTAGACTA
GCAATCTCTTCATTATTCTCTGCTATATATAAAACGGTGCTGTGAGGGAGGGGAAAA
GCATTTTTCAATATATTGAACTTTTGTACTGAATTTTTTTGTAATAAGCAATCAAGG
TTATAATTTTTTTTAAAATAGAAATTTTGTAAGAAGGCAATATTAACCTAATCACCA
TGTAAGCACTCTGGATGATGGATTCCACAAAACTTGGTTTTATGGTTACTTCTTCTC
TTAGATTCTTAATTCATGAGGAGGGTGGGGGAGGGAGGTGGAGGGAGGGAAGGGTTT
CTCTATTAAAATGCATTCGTTGTGTTTTTTAAGATAGTGTAACTTGCTAAATTTCTT
ATGTGACATTAACAAATAAAAAAGCTCTTTTAATATTAGATAA

…and here?

aaaa aaaa aaaa aaaa aaaa aaaa aaaa… “crystalline”

asmjgre fj sdjf s rege geoiie rgeasdffi… “amorphous”

…_ _ _ … … _ _ _ … …_ _ _ … … _ _ _ … “layered”

1001 110 11001 1111 10101 1 10010 101 1101 1 10010 10010 … “?”

If you are married or are a man and woman living together as if you are married you must claim jointly … “complex”

How might we even recognize something as being “complex”?

What about a “symbolic” Edge of Chaos?

the the the the the the the the the the the the the the the…. ordered

mercy proudly rush interrogative registered clansman therapeutic… disordered

Parameter to distinguish between the ordered and disordered states…
s – where:

Zipf's law may be stated mathematically as:

$$f(k; s, N) = \frac{1/k^s}{\sum_{n=1}^{N} 1/n^s}$$

where N is the number of elements, k is their rank, and s is the exponent characterizing the distribution. In the example of the frequency of words in the English language, N is the number of words in the English language and, if we use the classic version of Zipf's law, the exponent s will be equal to unity. $f(k; s,N)$ will then be the fraction of the time the kth most common word occurs.

In Hamlet (and more generally in natural language) s is about 1

So, natural language is on the “Edge of Chaos”!

Is that now an adequate description of Hamlet? That the frequency distribution of words is scale invariant with exponent s?

NO!

So what’s in Hamlet that isn’t in a “sandpile”?

Language from the point of view of a martian statistical physicist

$$< \alpha_i > \quad i \in a, b, ..., z$$

t is much higher frequency than x

$$< \alpha_i \alpha_j > \quad i, j \in a, b, ..., z$$

t and h are much more correlated than x and q; high peak in adjacent positions

$$< \alpha_i \alpha_j \alpha_k > \quad i, j, k \in a, b, ..., z$$

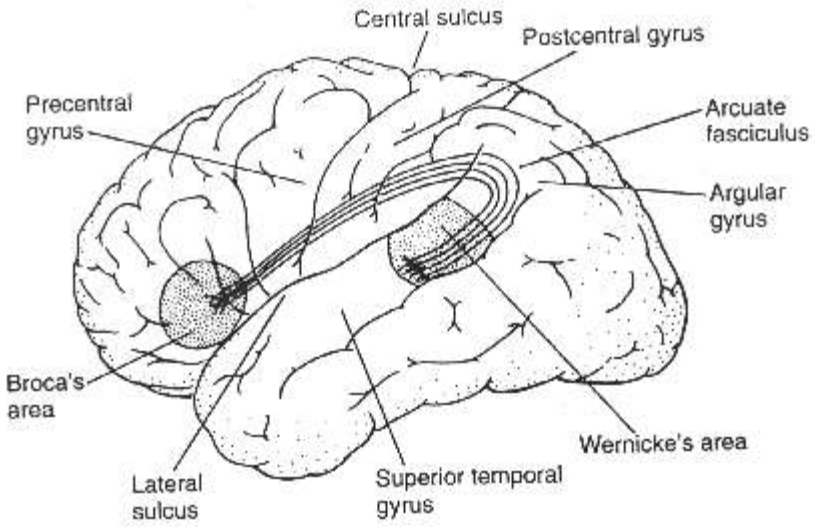
t, h and e are highly correlated in adjacent positions; detection of the "word" effective degree of freedom. Can then look at correlation functions between these new EDOF.

.

.

So Hamlet will show highly non-trivial correlation functions that show neither order nor disorder, but much more structure than Edge of Chaos. The correlation functions are our "structure detector/measurement device". But are statistical correlation detectors sufficient?

What's a better measurement device?

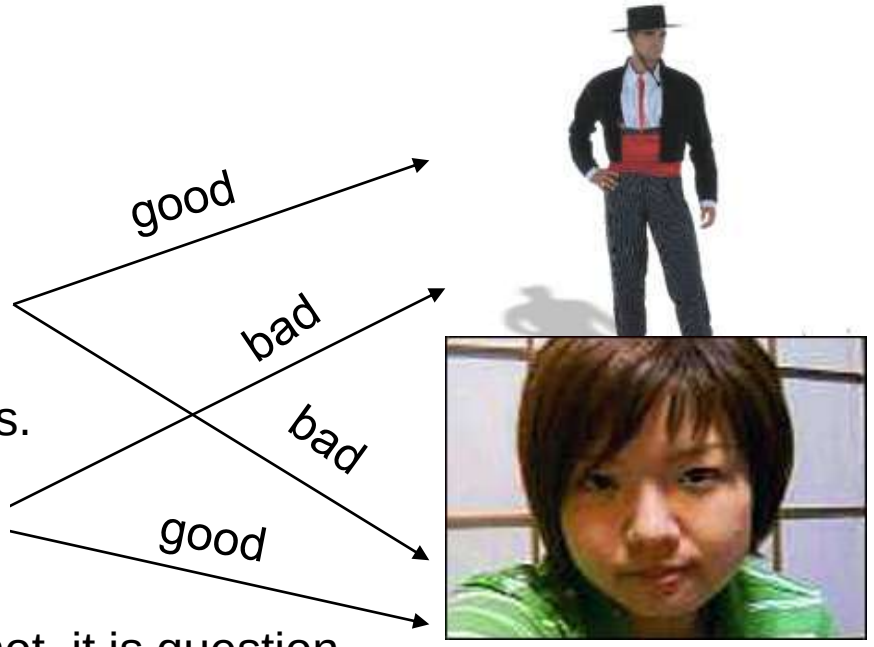


This measuring device certainly seems capable of measuring complexity.
Or does it…?

How good is your measuring apparatus?

- To be or not to be that is the question.
- Para ser o no ser que es la pregunta.
- Om te zijn of te zijn niet dat de vraag is.

あるためまたはないため質問である



- Because of a certain or because it is not, it is question.
- Because or it is not for the sake of, that having asked and being convinced.
- Being not to be for the sake of, or that that, you ask, are convinced.
- It is that without having for the sake of, or, you ask, are convinced.

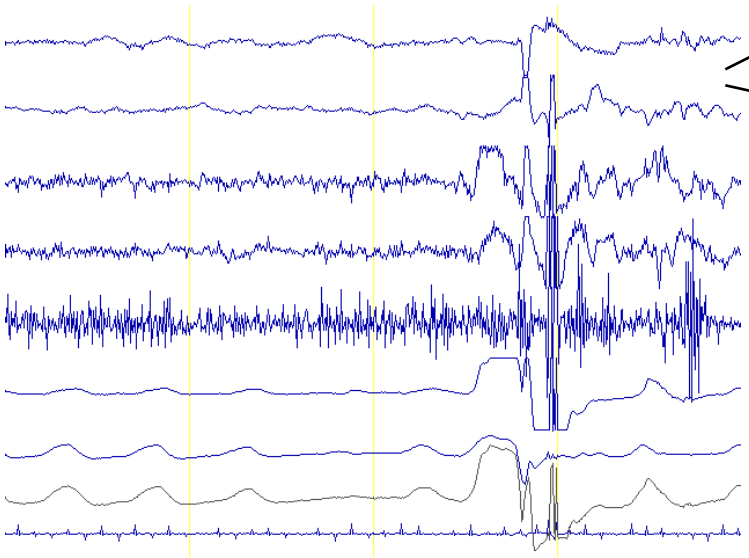
But is this any different than the physical world?

good

bad

bad

good



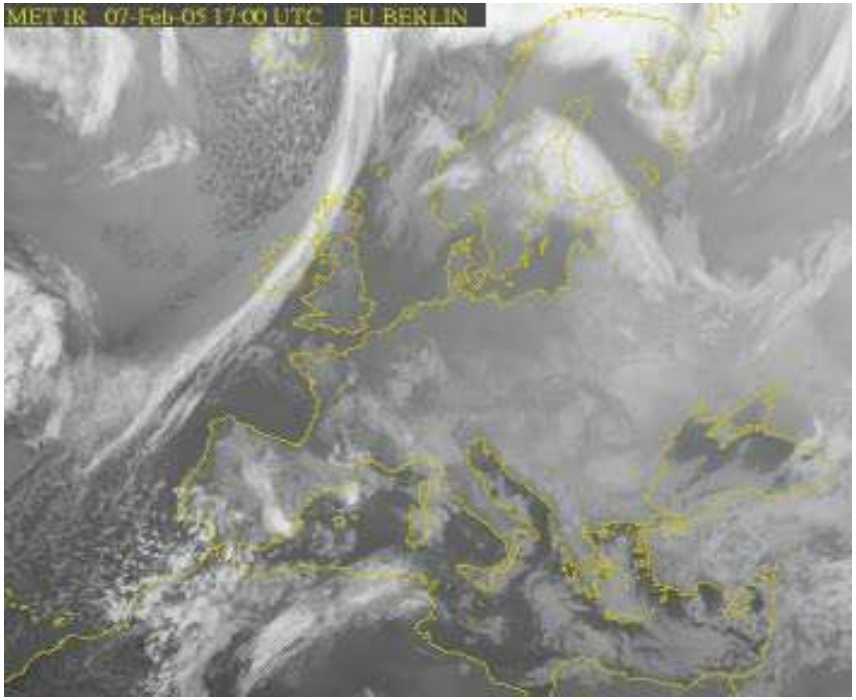
- So, is complexity more a property of a system **and** a measuring apparatus together rather than something intrinsic to a system itself?
- Different measuring devices measure it in different ways and some are more appropriate than others

How do we know if our measuring device is good? How do we distinguish between something that is complex but viewed with an inappropriate device versus something that is not complex?
In other words, do we know if we'd recognise it if we saw it?

So if symbolic complexity is a property of both a system and the measuring device, what about physical complexity in systems like…?

…the weather

47
49
50
50
51
…



Complex?

…or just complicated?

A **physical** phenomenon; underlying dynamics governed by Navier-Stokes equation (non-linear PDE)
Chaotic beyond 15 day horizon
"No" biological (human) component - Physics paradigms appropriate

Complexity – Subjective or Objective?

How Weather Affects Your Life	
Health	Health Forecast, Allergies, Skin Protection, Air Quality, Aches & Pains, Cold & Flu, Fitness
Travel	Travel Forecast, Business Traveler, Vacation Planner, Aviation
Driving	Driving Forecast, Interstate Forecast, Scenic Drives, Auto Advisor, Green Vehicles, Vehicle Safety
Events	Events Forecast, Sporting Events, Special Events
Recreation	Recreation Forecast, Golf, Boat & Beach, Outdoors, Ski
Home & Garden	Home & Garden Forecast, Home Planner, Lawn & Garden, Scotts Lawn & Garden Center, Schoolday
World	World Weather Forecasts & International Sites
News	News Center, Storm Watch, Tropical Update, Storm Stories, Road Crew
Weather Tools	My Page, Desktop, Email, Phone, PDA, Pager, My Site
Interact	Photo Gallery, Boards & Forums, Contact Us
Education	Weather Classroom, Dave's Dictionary, Weather Encyclopedia, Glossary, SafeSide, Rays Awareness
Multimedia	Video Forecasts
Shopping	The Weather Channel Store, Hot Offers and Cool Deals
TV - What's On	Storm Stories, Schedule, Road Crew, Personalities, Music, Forecast Earth
Mobile	Downloads, Messaging, PDAs

What's complex? The underlying phenomenon or our description of it?

The underlying phenomenon is not complex but its effects at the human level and our description of them are!

So what about something with a human component? Like…

…a stock market?

2086.66
2057.64
2075.06
2068.70
2062.41
2035.83
2047.15
…

Complex?

…or just complicated?

Geometric Brownian motion
Black-Scholes equation – Diffusion equation type PDE

Rational agents
Market efficiency
Equilibrium economics

Still very "physicsy" paradigms

Complexity – Subjective or Objective?

What's complex? The underlying phenomenon or just our description of it?

If markets are “efficient”, then they’re described by a “random” process and “predicting” the market seems to be then no different than counting elephants in the clouds or seeing people’s faces in a fire!

Financial markets: “Complex” Human Behavior → “Simple” Random Price Movements

Weather: “Simple” Navier-Stokes dynamics → “Complex” Human Behavior

So, what are we to make of all this?

- **Is it useful to distinguish between intrinsic (the system only) versus extrinsic (system and measuring apparatus)?**
- **I think so – can talk about correlations intrinsic to a system and correlations between a system and a measuring device**

Modeling complexity and complex systems

Consider the following “simple” dynamical model…

$$\mathbf{d}_i(t+\Delta t) = \sum_{j \neq i} \frac{\mathbf{c}_j(t) - \mathbf{c}_i(t)}{|(\mathbf{c}_j(t) - \mathbf{c}_i(t))|} + \sum_{j=1} \frac{\mathbf{v}_j(t)}{|\mathbf{v}_j(t)|}$$

Competition between effective repulsion and attraction between “particles”

$$\hat{\mathbf{d}}_i(t+\Delta t) = \mathbf{d}_i(t+\Delta t) / |\mathbf{d}_i(t+\Delta t)|$$

$$\mathbf{d}_i{}'(t+\Delta t) = \frac{\hat{\mathbf{d}}_i(t+\Delta t) + \omega \mathbf{g}_i}{|\hat{\mathbf{d}}_i(t+\Delta t) + \omega \mathbf{g}_i|}$$

Equation for “charged” particles following an external force vector g_i

Couzin, I.D., Krause, J., Franks, N.R. & Levin, S.A. (2005) *Nature,* **433**, 513-516.

Does this represent a “complex” system?

- In this mathematical model there are only two scales:
  - The “micro-” associated with individual fish and their typical distances
  - The “macro-” associated with the school or shoal itself (remember the “sandpile” on the Edge of Chaos)
- Saying shoaling is an “emergent” phenomenon is like saying boiling is an emergent phenomena

- So, we are using a non-complex model to describe a complex system
- The complexity is associated with a range of behaviors and functions
- The model only describes statistically one restricted aspect of this rich complexity
- Need a more complex model to describe more complex behavior

Moral: It's important to distinguish between a description of complexity and a non-complex description of a phenomenon or behavior associated with a complex system.

So, what properties should mathematical models have if they are to model complexity?

In biological, economic and social systems, organisms exhibit a rich array of (survival) **STRATEGIES (rules/models)**

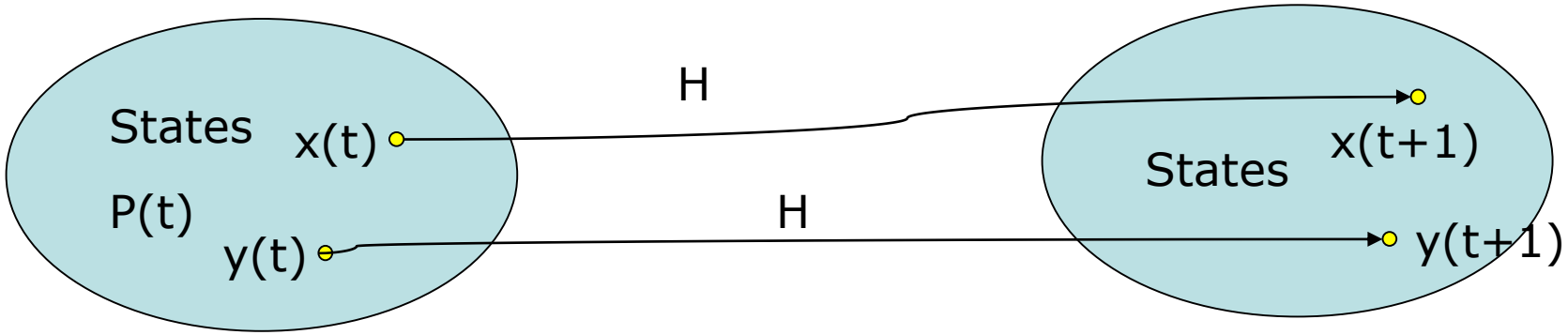
The dynamical state of an individual at t+1 depends on not only on the state of the individual and others at t but also on which strategy (update rule) is chosen at t, which in turn depends on the update rules of others at t

need to work in the space of states and strategies/rules/models - sounds like game theory, but …

We don’t a priori know what that space is!

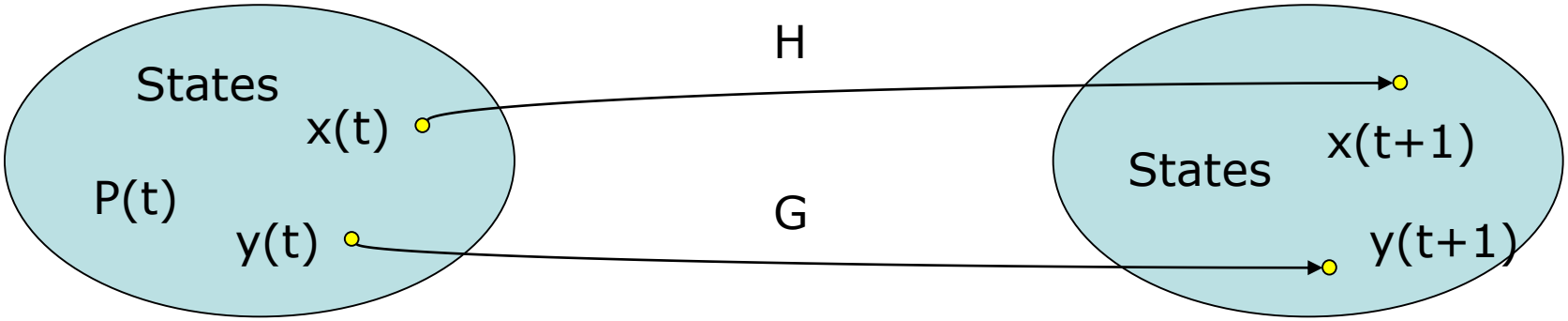
Also, the payoff/fitness for a strategy is RELATIVE not absolute – depends on the strategies used by others ⟹ a fixed fitness landscape is inappropriate; Fitness should be an emergent property. Imagine at the beginning of evolution specifying a priori the fitness of a lion!

Theoretical physics as it currently stands does not contain the mathematical and conceptual elements necessary to understand these issues…



Democratic evolution – one law for all (Physics)

$x(t+1)=H(x(t))$
$y(t+1)=H(y(t))$



Not all states are created equal (Not Physics)

$x(t+1)=H(x(t))$
$y(t+1)=G(y(t))$

G not equal to H ⇨ G(y(t)) not equal to H(y(t))

Theoretical Challenges for Modeling Complex Systems

- Develop frameworks within which one can work in the space of "laws" and states
- Understand what are "necessary" and "sufficient" conditions for complexity
- Statistical inference problems of observing complexity – can we speak the lingo?
- Work in a "game" where the rules change all the time and we don't know the payoffs
- Fitness as an emergent phenomenon
- Modularity – how to understand how different parts of a system can do different things then join together as "building blocks" to form more complex things
- Better understand the genotype-phenotype map
- Understand how to coarse grain (renormalization group) to see the emergence of effective degrees of freedom

Studying the following "experimentally" would help

- Develop systems that can do multi-tasking adaptively
- Develop systems where fitness is not specified
- Develop systems where modularity within a population emerges naturally – how do teams arise?
- Would these give us open ended evolution, i.e., continuous innovation?